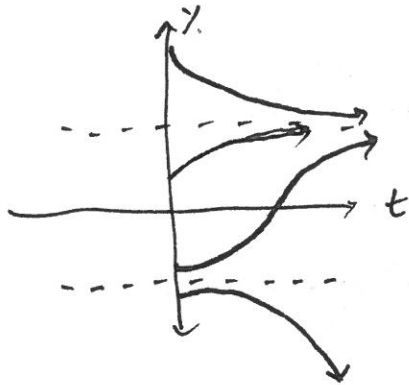
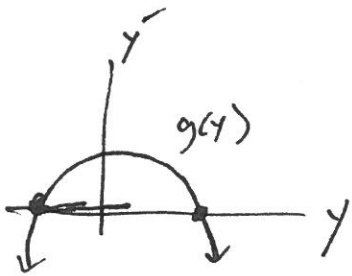


10.5 cont. Qualitative Theory

Sketching autonomous D.E.'s

$$y' = g(y)$$

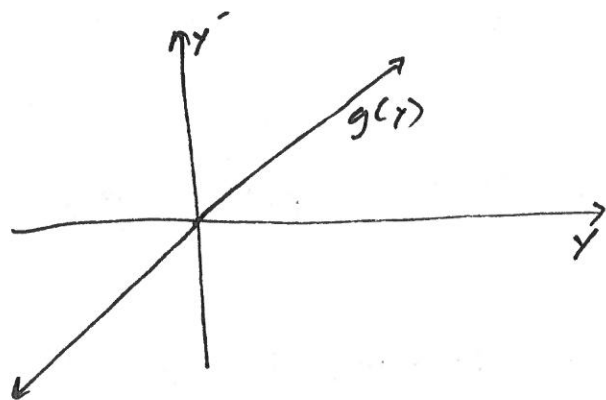
To sketch solutions to $y' = g(y)$ plot $g(y)$ on yy' plane



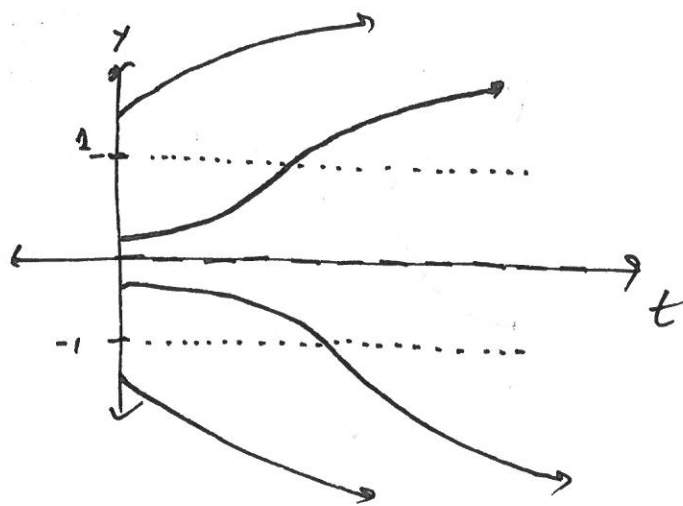
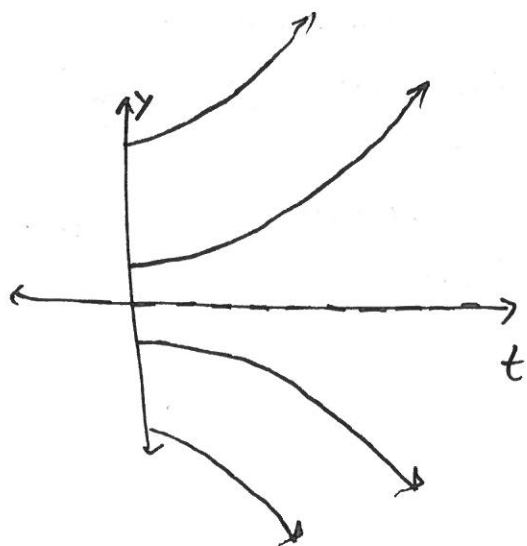
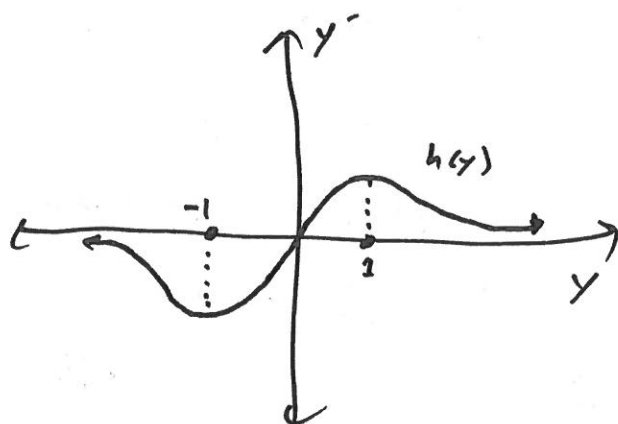
- Find constant solutions: $g(y) = 0$ (where $g(y)$ intersects y -axis) and plot on ty plane
- Find ~~constant~~ max & minimum of $g(y)$ corresponds with inflection & changes of solution curves.
- When $y' > 0$ then solution increasing
when $y' < 0$ solution decreasing

An example

$$y' = g(y)$$



$$y' = h(y)$$



• both have constant solutions

$$y = 0$$

• both are increasing when initial value is positive and decreasing when initial value is negative.

• ~~Right~~ Solutions of $y' = h(y)$ change inflection but solutions of $y' = g(y)$ do not

• what initial values give us strictly concave up solutions?

$$y(0) \leq -1$$

Plant Growth.

Suppose that a sunflower has a ~~mature~~ ^{mature} height M and its rate of growth is proportional to the product of its height and the difference between its mature height and current height

Recall ~~for~~ x & y are proportional if

$$y = kx \text{ for some } k.$$

Rate of growth: y'

Height: y

diff between current & mature height: $M - y$ (or $y - M$)

$$y' = k y (M - y)$$

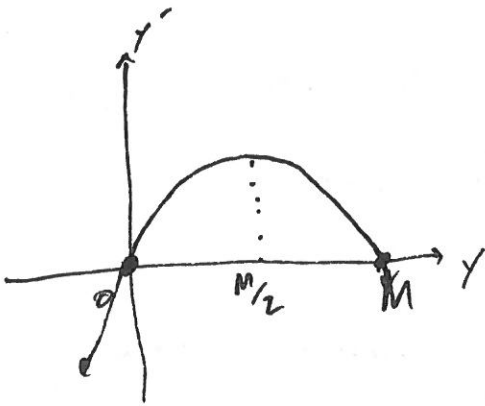
$$\text{or } y' = k y (y - M)$$

k is positive

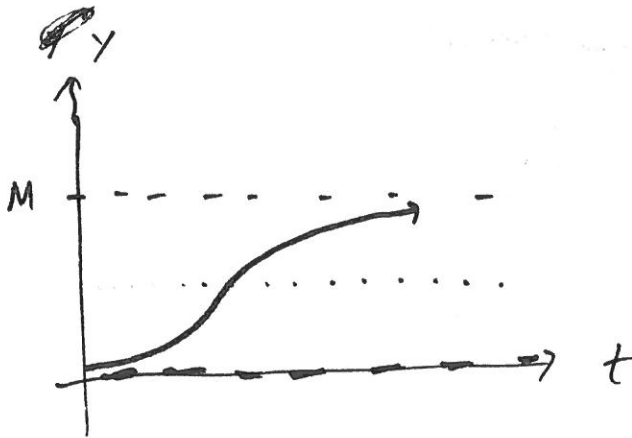
↑ This one is nicer.

k is negative

$$y' = ky(M - y)$$



Constant solutions
at $y = 0$
and $y = M$



$y' = ky(M - y)$ gives a logistic growth model

Uses to Model

- Population growth
- Disease Spread
- Rumor Spreading

Solution to $y' = ky(M - y)$
is

$$y = \frac{M}{1 + Ce^{-Mkt}}$$