7.1 Functions of Several Variables

Before: Functions of just one variable

Real life is often more complex...

Two variables: $f(x,y) = e^x(x^2+2y)$

$f(2,1) = e^2(4+2) = 6e^2$

$f(1,2) = e(1+4) = 5e$

Three variables: $f(x,y,z) = 5xy^2z$

$f(1,2,-3) = 5(1)(4)(-3)$

$= 20(-3) = -60$

Ex A store sells butter at \$4.50 per pound and margarine at \$3.40 per pound

Total revenue is then given by $f(x,y) = 4.50x + 3.40y$

where "x" is "pounds butter sold"
and "y" is "pounds margarine sold"

Other applications include:

- Temperature on Surface of Earth
  - latitude
  - longitude
  - time
- Drug Dosage
  - weight
  - Age
  - other medications
  - pill sizes

  ⋮

Cobbs-Douglas Production ~~Formula~~ function

- Costs of manufacturing come in two categories
  - Cost of labor "x"
  - Cost of Capital "y"

Economists have found that to find the production output i.e. "number of product manufactured" Often we can use a function that has the form

$$f(x,y) = C x^{A} y^{1-A}$$

where A & C are constants and $0 < A < 1$

Ex Suppose at a certain firm the number of goods produced follows

$$f(x,y) = 10 x^{3/4} y^{1/4}$$

- If we use 16 units of labor and 81 units of capital how much do we produce?

$$f(16,81) = 10(16)^{3/4}(81)^{1/4} = 10(2)^3(3) = 10(8)(3) = 240$$

- Suppose we swich our labor and capital costs, how much do we produce

$$f(81,16) = 10(81)^{3/4}(16)^{1/4} = 10(3)^3(2) = 10(27)(2) = 540$$

why should we expect this number to be bigger?

- Scaling up: what if we multiply our total labor and capital used by some multiple k?

$$f(81k, 16k) = 10(81k)^{3/4}(16k)^{1/4} = 10(81)^{3/4} k^{3/4} (16)^{1/4} k^{1/4}$$

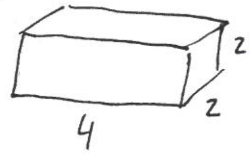
$$= 540 k^{3/4} k^{1/4}$$

$$= 540k = k \cdot f(81,16)$$

this constant Scaling called "constant returns to scale" ex/ Building an aquarium: 4 ft length, 2 ft high, 2 ft wide

Glass for the sides costs $\$3/ft^2$

Plastic for bottom costs $\$1/ft^2$



Bottom: $(4\cdot2)2 = \$16$

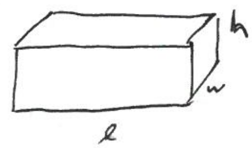
Sides: $(4\times2)3$ front $= \$2$
$(2\times2)3$ right $= \$1$
$(4\times2)3$ back
$(2\times2)3$ left

Total $= 16+24+24+12+$
$= \$88$

- What is the total cost?
- Write a function $f(l, w, h)$ if we were to change the dimensions to find the new cost.

front, right, back, left

4 sides: $l\cdot h,\ l\cdot h,\ w\cdot h,\ w\cdot h$



Total glass: $2lh + 2wh$

bottom: $l\cdot w$

Total plastic: $l\cdot w$

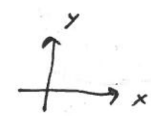
Total cost $= 3(2lh+2wh) + 2(lw)$

$= 6lh + 6wh + 2lw$

Graphing functions of more than 1 variable (well, 2 variables)

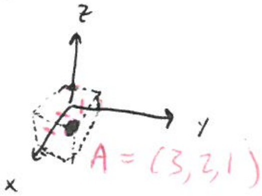
- How would we sketch $z = x^2 + y^2$?

For functions of 1 variable we plot on x, y axis



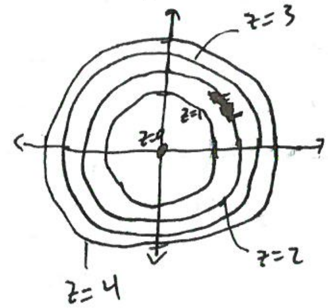
$y = x^2$:

For 2 variables x, y we will use 3-D set of axis



Use level curves to get shape: we will set one variable constant and look at plot

$z = x^2 + y^2$



gives a bowl shaped valley

$z = 0$ $\quad 0 = x^2 + y^2 \quad$ so $x = 0$, $y = 0$

$z = 1$ $\quad 1 = x^2 + y^2 \quad$ circle radius 1

$z = 2$ $\quad 2 = x^2 + y^2 \quad$ circle radius $\sqrt{2}$

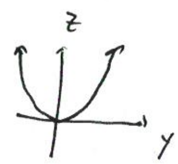
$z = 3$ $\quad 3 = x^2 + y^2 \quad$ circle radius $\sqrt{3}$

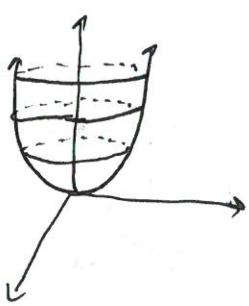
$z = 4$ $\quad 4 = x^2 + y^2 \quad$ circle radius 2

x-level curves:

$x = 0 \qquad z = y^2 + 0^2$

$z = y^2$





ex/ Topographic maps use level curves to tell us about elevation