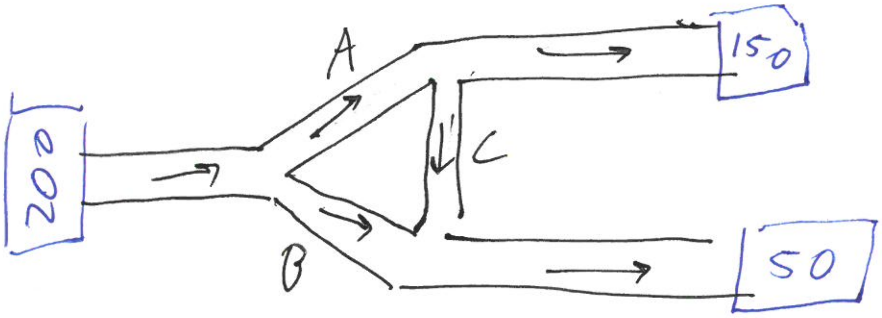
Day 11 + 10-02

3.3 continued

Last time: Traffic flow Problem:



x is the number of cars driving down Allen St. every hour

y " = Baker St.

z " = Coal St.

Traffic In = Traffic Out

200 cars enter intersection of Allen & Baker

so $\boxed{200 = x + y}$

150 cars leave intersection of Allen & Coal

$\boxed{x = z + 150}$

Bottom right: cars entering from Baker & Coal

cars leaving are the 50 being counted:

$\boxed{y + z = 50}$

$$\begin{cases} x + y = 200 \\ x - z = 150 \\ y + z = 50 \end{cases}$$

to write our matrix

$$\begin{bmatrix} 1 & 1 & 0 & 200 \\ 1 & 0 & -1 & 150 \\ 0 & 1 & 1 & 50 \end{bmatrix}$$

Row Reduce: $\begin{bmatrix} 1 & 0 & -1 & 150 \\ 0 & 1 & 1 & 50 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

∞ # of solutions

$$\begin{cases} x - z = 150 \\ y + z = 50 \end{cases} \Rightarrow \begin{cases} x = 150 + z \\ y = 50 - z \end{cases}$$

z is arbitrary.

Maximum Traffic on Baker Street?

What is the largest value y can have?

$y = 50 - z$

$z = 0$ b/c we can't have negative cars.

$y = 50$ → The maximum traffic / hour on Baker street.

Minimum possible traffic/hour on Allen?

$$x = 150 + z$$

if we want a minimum value for x here we want the smallest z possible.

$\Rightarrow z = 0 \quad \Rightarrow \quad x = 150$ is the minimum traffic on Allen.

Maximum possible traffic on Coal street?

z is arbitrary; look at $x = 150 + z$

$y = 50 - z$

what value for z would make the other equations "incorrect" in ~~an~~ this scenario?

$z = 50$ b/c $y = 50 - z$

$\Rightarrow$ Domain of z is $[0, 50]$

$0 \le z \le 50$

ex/ <u>Transportation</u>

NW has 20 extra cars

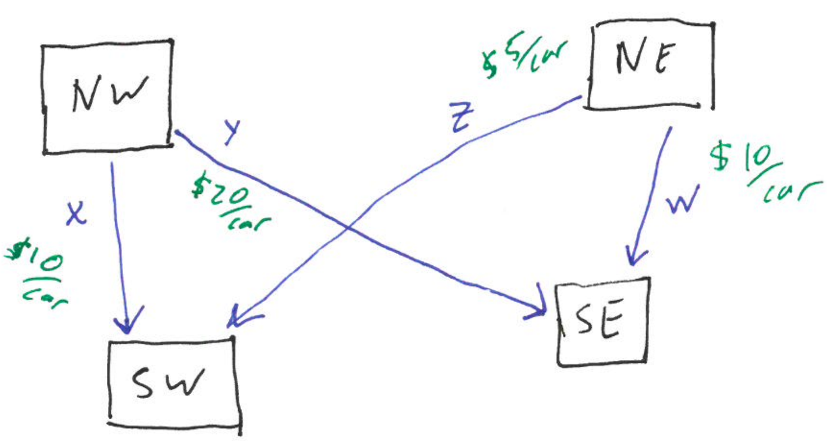
NE has 15 extra cars

SE needs 25 cars

SW need 10 cars.

total budget is

$475



20 cars will leave NW:

- $x+y=20$

15 cars will leave NE:

- $z+w=15$

25 cars will arrive in SW

- $x+z=25$

10 cars will arrive in SE

- $y+w=10$

Cost:

$$10x+20y+5z+10w=475$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 1 & 15 \\ 1 & 0 & 1 & 0 & 25 \\ 0 & 1 & 0 & 1 & 10 \\ 10 & 20 & 5 & 10 & 475 \end{bmatrix}$$

$\downarrow$ Row Reduce

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x=5$ NW → SW

$y=15$ NW → SE

$z=5$ NE → SW

$w=10$ NE → SE

Can we do for less money?

remove the last row that set the buget.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 1 & 15 \\ 1 & 0 & 1 & 0 & 25 \\ 0 & 1 & 0 & 1 & 10 \end{bmatrix} \xrightarrow{RR} \begin{bmatrix} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 1 & 25 \\ 0 & 0 & 1 & 1 & 15 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\cancel{x\ z\ z=5}$

$x - w = -5 \Rightarrow x = w - 5$

$y + w = 25 \Rightarrow y = 25 - w$

$z + w = 15 \Rightarrow z = 15 - w$

w is arbitrary (well not really)

$\text{Cost} = 10x + 20y + 5z + 10w$ plug in equations above

$\Rightarrow \text{Cost} = 10(w-5) + 20(25-w) + 5(15-w) + 10w$

$\text{Cost} = 525 - 5w$

How do we minimize the cost?

Make w large

→ largest it can be based on the problem is 15

so $\text{Cost} = 525 - 5(15)$

$= 450$

Quick Review on interpreting Row Reduced Matrices:

4 eq
4 unknowns

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \rightarrow \begin{matrix} x = 2 \\ y = 3 \\ z = 4 \\ w = 5 \end{matrix}$$

3 eq
2 unknowns
(x, y)

$$\begin{array}{ccc} x & y & = \\ \hline \end{array}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{cases} x = 3 \\ y = 4 \\ 0 = 0 \end{cases}$$

we run into ∞ solutions when we can't assign ~~a variable~~ every unknown an exact solution

2 eq
3 unknowns

$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & 5 \end{bmatrix} \end{array} \qquad \begin{cases} x + z = 4 \\ y - z = 5 \end{cases}$$

z is arbitrary

∞ - solutions.

$x = 4 - z$
$y = 5 + z$
$z = z$

2 eq
2 unknowns

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix} \qquad \begin{cases} x = 3 \\ 0 = 2 \end{cases}$$

this tells us there are no solutions.

Chapter 5 Linear Programing

Deals with optimization problems involving linear functions.

Section 5.1 Graphing Linear Inequalities

Recall important properties of inequalities:

$a \leq b$ "less than or equal to"

$a \geq b$ "greater than or equal to"

$a < b$ "strictly less than"

$a > b$ "strictly greater than"

ex $\quad 3 \leq 5 \qquad 4 \leq 4$

$\quad 3 < 5 \qquad 4 \geq 4$

Manipulating linear inequalities:

- for linear equations we could add & subtract to both sides without changing the equation
- This is also true for inequalities:

$$x \leq y \Rightarrow x + 3 \leq y + 3$$

$$\Rightarrow x - 10 \leq y - 10$$

doesn't change the inequality.

- We can also multiply or divide both sides of an inequality with a positive number and not change the inequality

$$x \leq y \Rightarrow \begin{matrix} 3x \leq 3y & x/10 \leq y/10 \\ 5x \leq 5y & \frac{3x}{2} \leq \frac{3y}{2} \end{matrix}$$

✱ • We can multiply or divide both sides of an inequality with a <u>negative</u> number, if we change the direction of the inequality.

✱ Multiplying an inequality by a negative # "flips" the inequality

$$x \le y \;=\; -3x \ge -3y$$

$$-\frac{x}{10} \ge -\frac{y}{10} \xrightarrow{\cdot 3} \frac{3x}{10} \le \frac{3y}{10}$$

$-3 < 2$ multiply both sides by -1

$\Rightarrow \quad 3 > -2$

$2 < 5 \quad \Rightarrow \quad -2 > -5$

How do we graph inequalities:

How do we graph linear inequalities?

A linear inequality (in 2 variables) is something we can put in the form

$$ax + by \le c \qquad \begin{matrix} \ge \\ < \\ > \end{matrix}$$

a, b, c are real constants

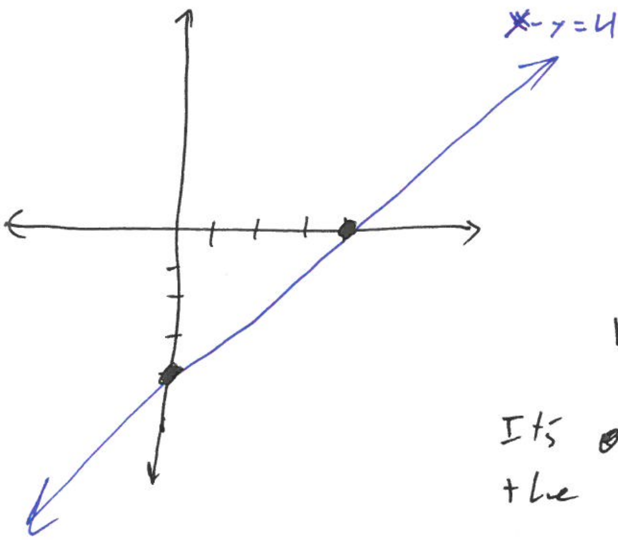
x, y are variables / unknowns.

ex/ $x - y \le 4$

<u>1st step</u> → ~~find the line~~ write as an equation and find the line representing the boundary

$x - y \leq 4$

$x - y = 4$ is a line



x-intercept at $y = 0$

$\Rightarrow \boxed{x = 4}$

y-intercept at $x = 0$

$-y = 4 \Rightarrow \boxed{y = -4}$

but when is $x - y \leq 4$?

It's on all the points on one side of the line.

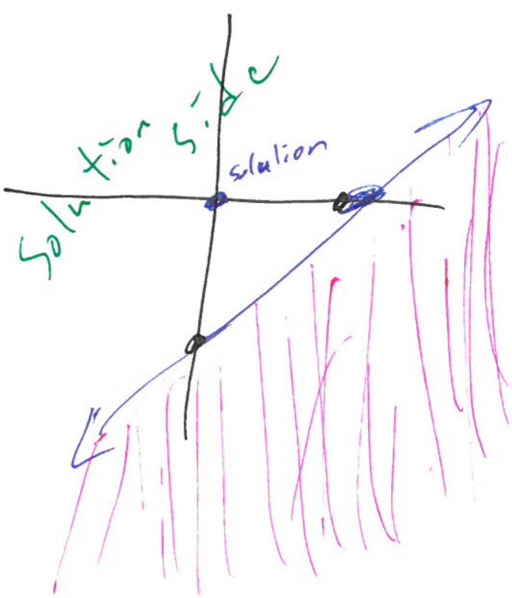
which side?

The easiest way to figure out is to pick a test point and see if it works

choose $(0,0)$ so $0 - 0 \leq 4$?

$0 \leq 4$ ✓

so since $(0,0)$ is a solution all points on that side we also a solution



★ Important

We will shade in the side that does not have solutions. Think of it as covering up the non-solutions.