~~Last time~~ Last time;

Linear Equations: $y = mx + b$

we will also write them in the form

$$Ax + By = C$$

So to write $y = mx + b$ in this form above

$$\begin{aligned} y &= mx + b \\ -mx & \quad -mx \end{aligned}$$

$\rightarrow \quad -mx + y = b$

ex/ $y = -\frac{3}{5}x + \frac{3}{10}$

$+\frac{3}{5}x \qquad +\frac{3}{5}x$

$\Rightarrow \left(\frac{3}{5}x + y = \frac{3}{10}\right) \cdot 10$

$\Rightarrow 2 \cdot 3x + 10y = 3$

$\Rightarrow 6x + 10y = 3$

3.1 Systems of two Equations with two unknowns.

- Given two "different" linear Equations with two unknowns (in this case our unknowns are usually x & y) we want to find values for our unknowns that will satisfy both equations.

ex/ Find two numbers whose sum 3 and whose difference is 1

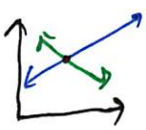
$$x + y = 3$$
$$x - y = 1$$

How do we solve these systems.

1.) Graphically : Both of these are linear equations
what does it mean for a point to "satisfy" a linear equation?
→ the point exists on the line

ex/ $y = x + 2$ $(2, 4)$ satisfies the line
$(3, 7)$ is not on the line and does not satisfy the equation.

Given two linear equations (two lines) they are both satisfied where they intersect.



- what if theyre parallel?
  → no solutions

- what if the lines are the same?
  → ∞ solutions.

<u>3 possibilities</u>

1.) Exactly one solution
2.) no solution
3.) Infinitely many solutions

ex/ $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$ → one solution

$\begin{cases} \frac{1}{2}x + y = 3/2 \\ 17x + 34y = 51 \end{cases}$ → ∞ solutions (multiply the first by 2 divide the second by 17)

$\begin{cases} \frac{2}{5}x + 2y = 5 \\ 2x + 10y = 7 \end{cases}$ → no solutions (solve for the slope by puting in $y = mx + b$)

2nd way to solve:

Algebraicly: (i.e. substitution & Elimination)

- substitution: Solve one equation for one of our unknowns
  → plug that into our second equation.

ex/

$x + y = 3$

$x - y = 1$

① solve for y

$x + y = 3$
$\quad -x \quad -x$

$\Rightarrow y = 3 - x$

plug into 2nd EQ

$x - (3 - x) = 1$

$x - 3 + x = 1$

$2x - 3 = 1$
$\quad +3 \quad +3$

$\frac{2x}{2} = \frac{4}{2}$

$\boxed{x = 2}$

$x + y = 3$

$2 + y = 3$

$\Rightarrow \boxed{y = 1}$

- Elimination*: Combine the two equations in ways that let us cancel one of the unknowns

$x + y = 3$

$x - y = 1$

idea: we want to find (x, y) so both equations are true
if $(x - y)$ equals (1) then if I add 1 to the right side of an equation it is ok to add $x - y$ to the left side

$x + y = 3$
$+(x - y) \quad +1$

$2x - 0y = 4$

$\Rightarrow 2x = 4$

$\Rightarrow \boxed{x = 2}$ plug into other eq's to get $\boxed{y = 1}$

* this is how solving systems with matrices works

why not just always use the graphical method?

ex/ $\begin{cases} 3x + 5y = 0 \\ 2x + 7y = 1 \end{cases}$ Desmos $(-0.455, 0.273)$

Algebraically: will need to multiply the equations ~~to~~ ~~eq~~ before we add

let's multiply the first by -2 and the second by 3

$$\begin{aligned} \Rightarrow -6x - 10y &= 0 \\ 6x + 21y &= 3 \\ \hline 0x + 11y &= 3 \end{aligned} \Rightarrow \frac{11y}{11} = \frac{3}{11} \Rightarrow \boxed{y = \frac{3}{11}}$$

plug into either eq

$$\boxed{x = \frac{-5}{11}}$$

note $\frac{3}{11} = 0.\overline{27}$

$-\frac{5}{11} = -0.\overline{45}$

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what does parallel lines and the same lines as with the graphical method look like algebraically.

ex/ $x - 3y = 5 \overset{\times 2}{\Rightarrow} 2x - 6y = 10$

$-2x + 6y = 8 \qquad -2x + 6y = 8$

$$\begin{aligned} 2x - 6y &= 10 \\ -2x + 6y &= 8 \\ \hline 0x + 0y &= 18 \end{aligned}$$

$\Rightarrow 0 = 18 \Rightarrow$ no solution!

called an <u>inconsistent system</u>

ex/ $x + y = 2 \overset{\times 2}{\Rightarrow} 2x + 2y = 4$

$2x + 2y = 4$

$$\begin{aligned} 2x + 2y &= 4 \\ 2x + 2y &= 4 \\ \hline 0 &= 0 \end{aligned} \Rightarrow \infty \text{ solutions}$$

called <u>redundant</u> or <u>dependent</u> system.

How do we write the solutions to a redundant system?

1. let either unknown be arbitrary

"let x be whatever"

then write the second unknown in terms of the first

"then what does y have to be?"

x is any real number $\Rightarrow$ $x + y = 2$

$y = 2 - x$

solutions are $(x, 2-x)$

or y could be any real number $(2-y, y)$

You run a movie theater where student tickets sell at \$4.50 and regular tickets for ~~\$9.00~~ \$9.00. So yesterday 700 tickets were sold and you made \$4,275. How many student tickets were sold & how many regular tickets?

x - # students tickets
y - # regular tickets

$x + y = 700$

$4.50x + 9y = 4275$

$x = 450$ $y = 250$

We are given a two digit # where the digits add up to 7.
If we switch the digits the number increases by 27 what is the number?

$\underline{x}\,\underline{y}$

$x + y = 7$

$10x + y + 27 = 10y + x$