Demand and Supply Models

Basic rule: Demand decreases as the price increases.

We will use q - (the quantity) of demand

we can measure demand by looking at sales data.

p - price of what we are selling

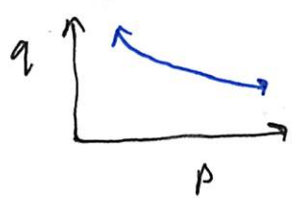
$q(p)$: will give us the demand as a function of price.

ex/ We are setting the price for tuition at a private school

$$q(p) = 77.8\,p^{-0.11} \text{ thousands of students}$$

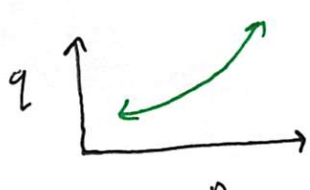
$$(200 \le p \le 2{,}000)$$ p - semester price in \$

Demand curve - The graph of our demand function



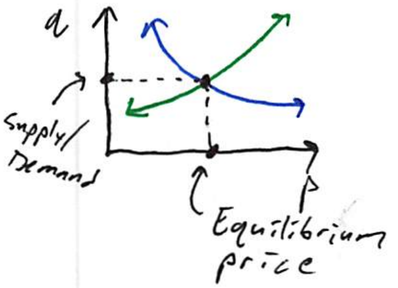
Supply function: express how much of an object we are willing to sell given the price.

generally supply increases as price increases.
we will also q to express supply.



As price ↑ ⇒ demand ↓ supply ↑

The Equilibrium Price occurs when demand and supply are the same.



How do we find this Equilibrium price?

- Analytically (set Demand & supply equal and solve for price)
- Approximate with software

ex/ You've designed a rad T-shirt that you're selling. We model demand by

$$q(p) = -20p + 800 \text{ shirts a day}$$

and supply by

$$q(p) = 10p - 100 \text{ shirts a day}$$

Find equilibrium price

$$-20p + 800 = 10p - 100$$
$$+20p \qquad\qquad +20p$$

$$\Rightarrow 800 = 30p - 100$$
$$+100 \qquad\qquad +100$$

$$\Rightarrow \frac{900}{30} = \frac{30p}{30} \Rightarrow p = 30$$

check: plug 30 into our equations

$$-20(30) + 800 = -600 + 800 = 200 \text{ t-shirts } ✓$$

check our other equation

$$10(30) - 100 = 300 - 100 = 200 \; ✓$$

Compound Interest

Idea behind compound interest for investing:

Your money makes and that money helps make more money

ex/ You invest \$100 into an investment account that gives a annual yeild of 7% with the interest compounded annually

Start: \$100 after one year we will add 7% of our \$100 to our ~~acco~~ total.

interest: $100(0.07) = \$7$ } two steps

$7 + 100 = 107$

to do it in one step $100(1 + 0.07) = 107$

year 2 now we're starting \$107 and have annual yeild

$107(1 + 0.07) = \$114.49$

Can we calculate our total holdings given any year in one step?

yes! $A(t)$ - Total money after t years

$$A(t) = P_0(1 + r)^t$$

P_0 - Principal (money we started with)

r - interest rate

Looking at our previous example

$$A(2) = \$100(1 + 0.07)^2$$
$$= 100(1 + 0.07)(1 + 0.07)$$
$$= 107(1 + 0.07)$$

what if our interest is compounded monthly

→ "compounded monthly" → our interest is reinvested monthly

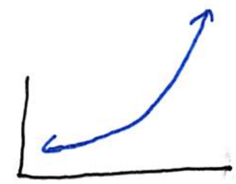
$$A(t) = P_0\left(1+\frac{r}{12}\right)^{12t}$$

r - is our annual rate

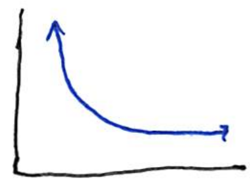
$r/12$ - is our monthly rate

exercise: formula for interest compounded weekly
~52 weeks in a year.

This is an example of exponential growth.



what about exponential decay?



example

Radioactive material decay

Half-Life: The amount of time it takes for half of our material to decay.

ex/ Carbon dating using carbon-14

Lets say that carbon-14 decays ~~at a rate of~~ such that after each year 99% remains.

$$A(t) = A_0(0.99)^t$$

Lets say we find a fossilized plant with 0.5g of Carbon in it. ~~We know from similar plants~~
We know it is 500 years old.
How much carbon was in it originally?

$$A(500) = A_0(0.99)^{500}$$

$$.5 = A_0(0.99)^{500} \Rightarrow A_0 = \frac{.5}{0.99^{500}} \approx 76g$$

1.3 Linear Functions

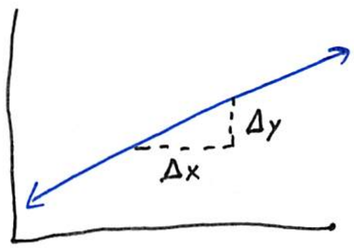
what is a linear function?

a function that can be expressed as

$f(x) = mx + b$ (↑ the same ↓) $y = mx + b$

m - slope

b - y - intercept



what is slope?

$\frac{\Delta y}{\Delta x}$ Δ means "change in"

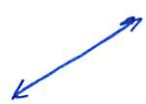
$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

the slope gives us the rate of change in y as we move along x.

How to compute slope?

Given two points (x_1, y_1) & (x_2, y_2) use $\frac{y_2 - y_1}{x_2 - x_1}$

ex/ $(2,3)$ & $(4,-3)$ $\frac{-3-3}{4-2} = -\frac{6}{2} = -3$



positive slope y ↑ as x ↑

negative slope y ↓ as x ↑

$(3,3)$ & $(158,3)$ $\frac{3-3}{158-3} = \frac{0}{\#} = 0$

$(4,9)$ & $(4,6)$ $\frac{6-4}{4-4} = \frac{-3}{0}$

How to calculate y-intercept?

$y = mx + \underline{b}$ — if we have a point, plug it in and solve for b

(x_1, y_1) plug in

$$y_1 = m x_1 + b$$
$$-mx_1 \quad -mx_1$$

$$b = y_1 - m x_1$$

ex

$m = 5$ $\quad (x_1, y_1) = (2, 4)$ $\quad b = 4 - 2(5)$
$= 4 - 10 = -6$

$m = 0$ $\quad (x_1, y_1) = (1002, 3242)$ $\quad b = 3242 - 0(\#)$
$b = 3242$