Section 1.2 Functions & Modeling

What is mathematical modeling?

Use mathematical terms to represent real-life scenarios or situations.

All mathematical models in general are wrong, but many are useful and are good approximations of reality.

ex/ This morning at 8am the tempature was 76°F
An hour later at 9am the tempature was 79°F

T - temperature
t - time in hours since 8am

$T(t) = 76 + 3t$

- using this model what will the temperature be at noon?

12 - 8 = 4 hours

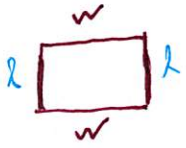
$T(4) = 76 + 3(4) = 76 + 12 = 88^\circ F$

- Using this model - what will the temperature be at midnight

$T(16) = 76 + 3(16) = 124^\circ F$

when is this model reasonable? (~~about~~ 8am)
within 4 hours of 8am

ex/ Fence in an enclosure with 100 ft of fencing.
We want to know what the area of this enclosure will be in terms of the width of the enclosure.



$P = 100 = 2l + 2w$

$A = wl$

step 1: solve for l

$$100 = 2l + 2w$$

$$\frac{100}{2} = \frac{2(l+w)}{2}$$

$$\Rightarrow 50 = l + w$$
$$\quad -w \quad -w$$

$$\Rightarrow l = 50 - w$$

step 2: plug in to Area equation

$A = wl$ plug in $l = 50 - w$

$$A = w(50-w)$$

- can only use widths in between 0ft and 50ft
- Still an approximation

Cost, Revenue, and Profit models

Lets look at the pricing models for Lime Bikes and Bird Scooters

MA 114 just let out and youre meeting a friend for brunch. at Big Eds

Bird scooter cost: $1 to start
+$0.15 / minute

Lime Bike cost: $1 per 30 minutes

Distance from here to Big Eds is 2.3 miles

t- is in minutes

by Google's estimate it will take 15 mintes

$$C_B(t) = 1 + 0.15t$$

$$C_L(t) = \begin{cases} 1, & 0 < t \leq 30 \\ 2, & 30 < t \leq 60 \\ 3, & 60 < t \leq 90 \\ \vdots \end{cases}$$

To make the pricing model for $C_L(t)$ nicer looking we will use a ceiling operator $\lceil x \rceil$ → gives next largest integer

ex/ $\lceil 2.3 \rceil = 3$ $\lceil 4 \rceil = 4$

floor operator is similar $\lfloor x \rfloor$ → gives the next smallest integer

- Lets reformulate our model for Lime Bike pricing.

$$C_L(t) = \left\lceil \frac{t}{30} \right\rceil \qquad C_B(t) = 1 + 0.15t$$

$$C_L(15) = \left\lceil \frac{15}{30} \right\rceil = \$1 \qquad C_B(15) = 1 + 0.15(15) = \$3.25$$

difference in cost is a Bacon Biscuit.

Meeting a friend in Chapel Hill? ~186 mins on the American Tobacco Trail

$$C_L(t) = \left\lceil \frac{186}{30} \right\rceil = \$7$$

$$C_B(t) = 1 + 0.15(186) = \$28.40$$

PSA: $5 bus pass for semester

Ex/ Cost - $ to run the business
Revenue - $ the business makes
Profit = Revenue - Cost
would like to positive profit or at least to break even.

Lets look at the operating costs of NCSU gym & recreation

$$C(x) = 100{,}000 + 160x - 0.2x^2$$

x - is the number of members
C(x) - cost in $/year

Cost models have a fixed cost and a variable cost

fixed cost: constant always present cost

variable cost: cost depends on an input/variable

for our model fixed cost: 100,000

variable cost: $160x - 0.2x^2$

↳ also called the marginal cost

total cost = variable cost + fixed cost

$R(x) = ?$ $168.85 + 92.50 + 23 + 27.5$ per student

$R(x) = 312x$ $R(x)$: Revenue in \$ / year

$$P(x) = R(x) - C(x)$$
$$= 312x - (100,000 + 160x - 0.2x^2)$$

how many students does it take for NCSU gym to break even?

1. Set $P(x) = 0$

$$\Rightarrow \quad 0 = 312x - (100,000 + 160x - 0.2x^2)$$
$$0 = 0.2x^2 + 152x - 100,000$$

How to solve for x? Use the Quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

this works and solves for x when $ax^2 + bx + c = 0$

plug these in to the formula (or computer)

$x = 192.839$ students

$= 193$ students