Last Day!

Warm up: A bag has
2 red marbles
3 green marbles
1 pink marble
4 yellow marbles
3 orange marbles

We grab 3 at random.
What is the probability we grab all the red marbles given we've grabbed the pink marble?

A: we grab both reds

B: we grab the pink marble

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ need to find $P(A \cap B)$ and $P(B)$

$P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{C(13,3)} = \dfrac{1}{286}$

$C(13,3) = \dfrac{13!}{10!\,3!} = \dfrac{13 \cdot 12 \cdot 11}{3 \cdot 2 \cdot 1} = 13 \cdot 2 \cdot 11$

$= 2 \cdot 143 = 286$

$P(B) = \dfrac{n(B)}{286} = \dfrac{C(12,2)}{286} = \dfrac{66}{286}$

$P(A \mid B) = \dfrac{1/286}{66/286} = \dfrac{1}{66}$

Final: Tuesday Cumulative

Course Evals Close Monday

7.5 continued

Independence

Define Mutually Exclusive events,

Events that cannot both happen

ex/ Roll 2 die (red & green)

A: The red die is odd

B: The red die is a 6

Going back to our ad for a pokemon game:

	Saw Ad	Did not See ad	total
Purchased	100	200	300
Did not Buy	200	1500	1700
Total	300	1700	2000

was the ad effective?

A: people bought game

B: people who saw the ad

To answer our question we will compare

$P(A)$ with $P(A|B)$

Probability someone bought game

probability someone bought game given they saw ad.

$$P(A) = \frac{300}{2000} = .15$$

$$P(A|B) = \frac{100}{300} = .33$$

So since $P(A \mid B) > P(A)$

it looks the ad was effective: People who saw the ad were more likely to buy the game.

<u>Case 1</u> from above: $P(A \mid B) > P(A)$

ad was effective (positive correlation)

<u>Case 2</u> $P(A \mid B) < P(A)$

ad backfired. (negative correlation)

A and B are dependent on each other

<u>Case 3</u> $P(A \mid B) = P(A)$

A and B are not related there was no difference.

<u>Independent</u>

If A & B are <u>independent</u> events they do not influence each other.

and $P(A \mid B) = P(A)$

<u>Recall:</u>

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

independent events

$$P(A) = \frac{P(A \cap B)}{P(B)}$$

$$\boxed{P(A)P(B) = P(A \cap B)}$$

independent events only

$$P(B) = \frac{P(A \cap B)}{P(A)} = P(B \mid A)$$

- How to test for independent events?

If $P(A)P(B) = P(A \cap B)$ independent

If $P(A)P(B) \neq P(A \cap B)$ not independent.

ex/ Roll ~~a~~ a red and green die

A: Red die is even

B: The dice are either both even or both odd

Are A & B independent?

$P(A) = \frac{1}{2}$

$P(B) = \frac{1}{2}$

$P(A \cap B)$

so $A \cap B = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6), \}$

$n(A \cap B) = 9$

$n(S) = 36$

so $P(A \cap B) = \frac{9}{36} = \frac{1}{4}$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ ✓ A & B are independent.

ex Roll a red and green die:

A: Red die is > 4

B: The dice add to 7

$P(A) =$ ~~~~ red die can be 5 ~~~~ or 6

~~~~ $n(A) = 12$

$P(A) = \frac{12}{36} = \frac{1}{3}$

$P(B) = \frac{1}{6}$

$P(A \cap B) =$ $\cancel{n(A \cap B) =}$ $A \cap B = \{(5,2), (6,1)\}$

$n(A \cap B) = 2$

$P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$ A and B are independent.

---

ex 50% chance of rain in New York - A

30% chance of rain in Honolulu - B

what is is the probability that it rains in both cites? Assume these events are independent.

$P(A)P(B) = P(A \cap B)$

$.5 \times .3 = .15$

☆ what is the probability it rains in at least one city?
~~~~

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= .5 + .3 - .15$$
$$= .65$$

Be careful with wording. What is the difference

if we are flipping two coins

- "there are two heads" = ~~two coin flips are dependent~~
- "the second coin is heads" = ~~two coin~~

→ result is dependent on both outcomes

→ result only depends on only 2nd.

If the probability of there being a bomb on an airplane is 0.000 001

Then the probability of there being two bombs on an airplane is (.000 01)(.000 01)

= .000 000 000 001

So we should always carry a bomb with us on airplanes.

False: A: probability you bring a bomb
B: " = someone else does

~~P(A~~ $P(B \mid A) = P(B)$

ex/ Are these two events independent?

Roll two distinguishable die.

A: Sum of our dice is 6

B: Both dice are odd

$$A = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$$

$$B = \{(1,1),(1,3),(1,5),\\ (3,1),(3,3),(3,5),\\ (5,1),(5,3),(5,5)\}$$

$$A \cap B = \{(1,5),(3,3),(5,1)\}$$

$$P(A) = \frac{5}{36}, \quad P(B) = \frac{9}{36} = \frac{1}{4}, \quad P(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

$$= \frac{5}{36} \cdot \frac{1}{4} \neq \frac{1}{12}$$

A and B are not independent.