Warm up: Given the following table of probabilities

Outcome	a	b	c	d	e
Probability	.1	.05	.6	.05	.2

$P(a)+P(b)+P(c)+P(d)+P(e)=1$

$.1+.05+.6+.05+P(e)=1$

$.8+P(e)=1$

compute: a.) $P(\{a,c,e\})$

$= P(a)+P(c)+P(e) = .1+.6+.2 = .9$

b.) $P(E \cup F)$ where $E=\{a,c,e\}$
$F=\{b,c,e\}$

$= P(E)+P(F)-P(E\cap F)$

$= .9+.85-.8 = .95$

or see that $P(E\cup F) = P(\{a,c,e,b\})$

$= P(a)+P(c)+P(e)+P(b)$

$= .95$

c.) $P(E')$

$= P(\{b,d\}) = P(b)+P(d) = .1$

d.) $P(E\cap F)$

$= P(\{c,e\}) = P(c)+P(e) = .6+.2 = .8$

ex/ Four distinguishable fair coins.
If we flip these four coins what is the probability that we will have <u>at most</u> one tail?

- So events we are interested in:

$E = \{HHHH, THHH, HTHH, HHTH, HHHT\}$

$n(E) = 5$

- How many possible outcomes are there for our expirement of flipping 4 coins?

16 different ways to flip 4 distinguishable coins:

1st coin : 2 options
2nd coin : 2 options
3rd : 2 options
4th : 2 options

$2\cdot2\cdot2\cdot2 = 2^4 = 16$

- Since these are fair coins each outcome will be equally likely so the probability of flipping 4 coins and getting at most one tail is

$$\boxed{\frac{5}{16}}$$

ex/ Make a table of probabilities for the expirement of rolling 2 distinguishable fair dice and taking their sum.

Sum	1	2	3	4	5	6	7	8	9	10	11	12
# of ways to get sum	0	1	2	3	4	5	6	5	4	3	2	1
~~Probabilit~~ Probability	0	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

note: all of these rolls are equally likely.

36 total ways to roll 2 distinguishable dice.

ex/ Probability to roll a 7 $= \dfrac{\text{\# ways to roll a 7}}{\text{\# of ways to roll the dice}}$

7.4 Counting & Probability

This section puts together counting and finding probabilities

In this section we will be dealing with equally likely outcomes:

$$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{\#\ \text{of favorable outcomes}}{\#\ \text{of total outcomes}}$$

ex/ We have a bag with four red marbles and two green marbles.
If we grab 3 at random, what is the probability we will grab both green marbles?

- we already know how to count the number of ways this can happen, so now we just need to use the formula above after counting.

what is $n(S)$, how many ways can we grab 3 marbles? "6 choose 3" or $C(6,3)$

$$\frac{6!}{(6-3)!\,3!} = \frac{6\cdot5\cdot4\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{3\cdot2\cdot1\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = \frac{{}^{2}\cancel{6}\cdot5\cdot\cancel{4}^{2}}{\cancel{3}\cdot\cancel{2}\cdot1} = 2\cdot5\cdot2 = 20$$

- 20 possible ways to take 3 marbles from a bag of 6.

- $n(E)$? How many ways are there to take 2 green marbles and 1 red marble from our bag?

Step 1: pick 2 green $\quad C(2,2)$

Step 2: pick 1 red $\quad C(4,1)$

$C(2,2)\cdot C(4,1)$

$1\cdot4 = 4$ ways

So $n(E) = 4$ $\quad P(E) = \frac{4}{20} = \frac{1}{5} = 20\%$

$n(S) = 20$

Poker $\quad$ 5-card poker, dealt 5 cards

what is the probability that we are dealt a full house?

- How many ways can we be dealt 5 cards?

$$C(52, 5) = 2,598,960$$

- How many ways can we get a full house?

step 1: choose a denomination $\quad C(13,1) = 13$

step 2: Pick 3 cards from that denom $\quad C(4,3) = 4$

step 3: Choose a differend denom. $\quad C(12,1) = 12$

step 4: Pick 2 cards from it. $\quad C(4,2) = 6$

$$n(E) = 13 \cdot 4 \cdot 12 \cdot 6 = 3,744$$

Probability of a full house is $\frac{3,744}{2,598,960} \approx .0014$ or 0.14%

How many ways to be dealt two pair

a two pair is 2 of one denomination A-K
2 of a different denomination
1 other card.

ex ~~2~~ 2 hearts
2 diamonds
6 clubs
6 spades
Q hearts

This is a bit different than how we did the full house

Step 1: Pick two denominations $C(13,2)$

Step 2: Choose 2 cards from first $C(4,2)$

Step 3: Choose 2 cards from the second $C(4,2)$

Step 4: Choose a ~~card from~~ third denom $C(11,1)$

Step 5: Pick one from the third denom. $C(4,1)$

$$C(13,2)\cdot C(4,2)\cdot C(4,2)\cdot C(11,1)\cdot C(4,1) = 123{,}552$$

So the probability of getting two pair

$$= \frac{123{,}552}{2{,}598{,}960} \approx 0.0475 = 4.75\%$$

ex Powerball lottery:

Choose 5 numbers from 1 to 69 white numbers

Choose 1 number from 1 to 26 powerball number

What is the probability of hitting the jackpot: match all 6 numbers correctly.

note: order does not matter.

Step 1: choose 5 ~~from~~ 69 $C(69,5) = 11{,}238{,}513$

Step 2: choose our power ball # $C(26,1) = 26$

$(11{,}238{,}513) \times 26$ = gives the # of ~~ways~~ different lottery tickets there are

$= 292{,}201{,}338$

only 1 way to win the jackpot

Probability of hitting the jackpot $= \dfrac{1}{292{,}201{,}338} = 0.000000342\%$