11-15

Warm-up:

In a survey of 570 Latin music downloads

350 were regional
135 were pop-rock
65 were tropical
20 were urban

Find the relative frequency for:

a.) A music download was regional

b.) Either tropical or urban

c.) Not urban

a.) $\frac{\text{\# of regional downloads}}{\text{\# of downloads}}$

$= \frac{350}{570}$

b.) $\frac{\text{\# tropical} + \text{\# urban}}{\text{total}} = \frac{65+20}{570}$

c.) $\frac{\text{\# regional} + \text{\# tropical} + \text{\# pop-rock}}{\text{total}} = \frac{350+65+135}{570}$

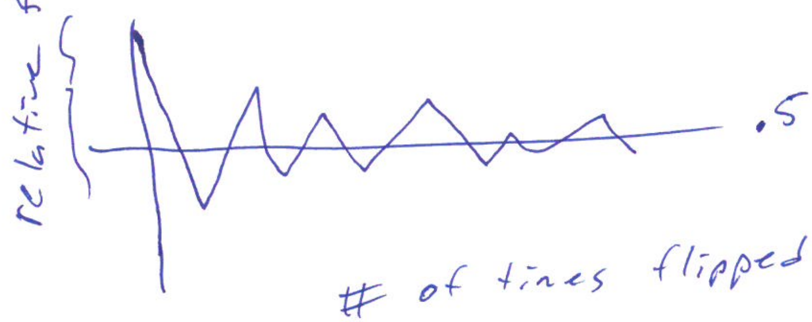
or $1 - \frac{\text{\# urban}}{\text{total}} = 1 - \frac{20}{570} = \frac{570}{570} - \frac{20}{570} = \frac{570-20}{570}$

7.3 Probability & Probability Models

Last time we found the relative frequency of flipping a coin 100 times and getting heads.
.53 last time.

- But what is the actual probability?

Flip a coin 100 times and make a graph of our relative frequency



Flip it 100,000 times

Probability distributions are a way to model specific events. Can think of it as if we N to be a larger and larger # approaching ∞

Important facts

A probability distribution for outcomes in our sample space (set of outcomes)

$$S = \{ s_1, s_2, \dots, s_n \}$$

$P(s_i)$ is the probability of that outcome

1.) $0 \leq P(s_i) \leq 1$ ~~no prob~~ nothing has less than 0% chance or more than 100% chance

2.) $P(s_1) + P(s_2) + P(s_3) + \dots + P(s_n) = 1$

100% probability that an outcome in our list will happen.

3.) To find the probability of an event E add up the probabilities of the outcomes in E

ex Rolling a fair die. $S = \{1, 2, 3, 4, 5, 6\}$

Outcomes	1	2	3	4	5	6
Probability	$P(1) = \frac{1}{6}$	$P(2) = \frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

1.) $P(3) = \frac{1}{6}$ $\quad 0 \leq \frac{1}{6} \leq 1$

2.) $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) =$

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$

Probability of rolling an even number?

Event: rolling an even number

$E = \{2, 4, 6\}$

3.) $P(2) + P(4) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

This is a probability model for equally likely outcomes.

$$P(E) = \frac{n(E)}{n(S)} = \frac{\#\text{ outcomes in } E}{\text{total }\#\text{ of outcomes}}$$

in the last example $n(E) = 3$

$n(S) = 6$

so $P(E) = \frac{3}{6} = \frac{1}{2}$

* This will not work if we used a weighted die of other ~~event where~~ expirement where the outcomes are not equally likely

ex Toss a fair coin 3 times. (keeping track of order)

eight outcomes.

ex $HHT \neq THH$

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

what is the probability that we get exactly 2 heads?

what are out comes in this event?

$E = \{HHT, HTH, THH\}$

$n(S) = 8$

$n(E) = 3$

$P(E) = \frac{3}{8}$

ex/ Roll a pair of fair dice. (distinguishable)

what is the probability we get doubles

$n(S) = 36$

Step 1: Roll first die - 6
Step 2: Roll 2nd die - 6

$6 \times 6 = 36$

How large is the set of doubles?

$E = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$ - 6 outcomes

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

What about ~~ind.~~ indistinguishable dice?

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,3), (3,4), (3,5), (3,6),$$
$$(4,4), (4,5), (4,6),$$
$$(5,5), (5,6),$$
$$(6,6)\}$$

$n(S) = 21$ different outcomes.

what is the probability of each outcome?

★ Will have to use Chapter 6 to answer this question.

How many different ways can we roll a $(1,1)$?

Only 1 way to roll a (1, 1)

How many ways to roll a (1, 2)?

How many options for the first die?

can be a 1 or a 2. 

Second die has to be a 1 if the first were a 2, and has to be a 2 if the first was a 1 

2 ways to roll a (1, 2)

Then we have twice as large a chance of rolling a (1, 2) than rolling a (1, 1)

So probabilities for these out comes are NOT equally likely. we <u>cannot</u> use $\frac{n(E)}{n(S)} = P(E)$

Ex/ We have a weighted die that is 3 times as likely to roll a 6 than any other number

x - probability of a 6

y - probability of any one other number ex/ 2

$3y = x$ also $P(1) + P(2) + \dots + P(6) = 1$

$y + y + y + y + y + x = 1$

$3y = x$

$5y + x = 1$ $\Rightarrow$ $5y + 3y = 1$ $\Rightarrow$ $y = \frac{1}{8}$

$x = \frac{3}{8}$

Outcomes	1	2	3	4	5	6
Probabilities	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{8}$

What is the probability of rolling an even number?

Can't do $\frac{3}{6}$ b/c outcomes are not equally likely.

$$P(2) + P(4) + P(6) = \frac{1}{8} + \frac{1}{8} + \frac{3}{8} = \frac{5}{8}$$

What about probabilities of unions of events?

ex/ E: roll a double

F: at least one die is odd

Recall from Chapter 6

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

works same way for probabilities

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Complements, Unions, intersections, all transfer the same way.

ex/

A 60% chance of rain

B 30% chance of high winds

C 10% chance of both

what is the probability of neither happening?

Hint: use De Morgans law

$$P(A' \cap B') =$$

$$P((A \cup B)')$$