11-8

Warm up: How many five-letter sequences are there that use the letters q, u, a, k, e, s each at most once?

ordering 5 elements from a set of 6

$$P(6,5) = \frac{6!}{(6-5)!} = \frac{6\times5\times4\times3\times2\times1}{1} = 720$$

ex The Boston Marathon in 1996 had 36,748 runners. How many ways are there to order first, second, third places.

ordering 3 elements from a set of 36,748

$$P(36{,}748, 3) = \frac{36748!}{(36748-3)!}$$

$$= 36748 \times 36747 \times 36746$$

What if order does not matter to us?

NC state Basketball team has 14 players on the roster.

- How many different way are there to have 1 v. 1 matches between the players?

Picking 2 out of 14.

Our first guess might be $P(14,2) = \frac{14!}{(14-2)!}$

$= 14 \times 13$

Not quite right. Why?

order does not matter

for example a 1 on 1 game between

C.J. Bryce vs. Devon Daniels

is the same as a 1 on 1 game between

Daniels vs. Bryce

$14 \times 13 = 182$ but how many redundancies are there?

for each match we have a second redundant match

so there are twice as many listed as we need.

$\Rightarrow \frac{182}{2} = 91$

During a game how many different ways are there to have 5 players on the court?

note: a, b, c, d, e

same as b, a, e, c, d

Lets start with number of Permutations

$P(14,5) = \frac{14!}{(14-5)!}$

How many duplicates in this list?

- Ans: 5! - the number of ways we can order 5 players.

Need to divide by 5!

$$\frac{14!}{(14-5)!\,5!}$$ total number of ways we can have 5 players on the court.

$$= \frac{14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1\quad(9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1)}$$

$$= \frac{14\cdot13\cdot\cancel{12}\cdot11\cdot\cancel{10}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = \boxed{14\cdot13\cdot11}$$

- An ~~ord~~ ordered list of ~~set~~ elements in a set is a <u>Permutation</u>
- An an unordered list of elements in a set is a <u>Combination</u>

$$P(n,r) = \frac{n!}{(n-r)!} \qquad C(n,r) = \frac{n!}{(n-r)!\,r!}$$

for combinations we will often hear $C(n,r)$ said aloud as "n choose r"

ex Calculate $C(11,3)$ and $C(11,8)$

$$11 \text{ choose } 3 = \frac{11!}{(11-3)!\,3!} \qquad 11 \text{ choose } 8 = \frac{11!}{(11-8)!\,8!}$$

$$= \frac{11!}{8!\,3!} \quad = \quad = \frac{11!}{3!\,8!} = 165$$

Think about it: The number of way to choose 8 from 11 is the same as <u>not</u> choosing 3 from 11

$$C(n,r) = C(n, n-r)$$

$$\frac{n!}{(n-r)!\,r!} \qquad \frac{n!}{(n-(n-r))!\,(n-r)!} = \frac{n!}{r!\,(n-r)!}$$

ex/ Lotto

- Range of 1-55 to pick from and we will pick 6 unordered numbers.
- If we wanted to guarantee a win and buy all the lottery tickets possible $1 will get you two tickets, how much money will you spend?

Answer: 55 choose 6 $= C(55,6)$

$$= \frac{55!}{(55-6)!\,6!} = 28{,}989{,}675 \text{ tickets.}$$

two tickets cost $1 so

$$\frac{28{,}989{,}675}{2} = \$14{,}494{,}838$$ to buy all the possible combinations.

ex/ Picking Marbles from a bag.

we have a bag with
3 red marbles
3 blue marbles
3 green marbles
2 yellow marbles

a.) How many sets of 4 marbles are possible?

Total # of marbles is 11

so 11 choose 4 gives the answer $\frac{11!}{(11-4)!\,4!} = 330$

b.) How many sets of 4 are there so that each marble is a different color?

step 1: Pick a red marble $C(3,1) = 3$
2: " " blue " " " $= 3$
3: " " green " " $= 3$
4: " " yellow " " $C(2,1) = 2$

$3 \times 3 \times 3 \times 2 = 54$

c.) How many sets of 4 marbles where at least 2 are red?

Either we will have 2 red marbles
or 3 red marbles

Alternative 1:
2 red
Step 1: choose 2 red
step 2: choose 2 non-red

Alternative 2:
3 red
step 1: Choose 3 red
step 2: Choose 1 non-red

$$\left(\begin{matrix}\text{\# ways to}\\ \text{choose 2 red}\end{matrix}\right)\left(\begin{matrix}\text{\# ways}\\ \text{choose 2 non-red}\end{matrix}\right) + \left(\begin{matrix}\text{\# ways to}\\ \text{choose 3 red}\end{matrix}\right)\left(\begin{matrix}\text{\# of ways to}\\ \text{choose 1 non-red}\end{matrix}\right)$$

$$[C(3,2)][C(8,2)] + [C(3,3)][C(8,1)]$$

$$3 \times 28 + 1 \times 8 = 84 + 8 = 92$$

d.) How many sets of 4 where none are red, but at least one is green?

Alternative 1: 1 green

Step 1: choose 1 green : $C(3,1) = 3$

Step 2: choose 3 non-red : $C(5,3) = 10$

Alternative 2: 2 green

Step 1: Choose 2 green : $C(3,2) = 3$

Step 2: Choose 2 non-red $C(5,2) = 10$

Alternative 3: 3 green

Step 1: Choose 3 green $C(3,3) = 1$

Step 1: choose 1 non-red $C(5,1) = 5$

$$3 \times 10 + 3 \times 10 + 1 \times 5 = 65$$