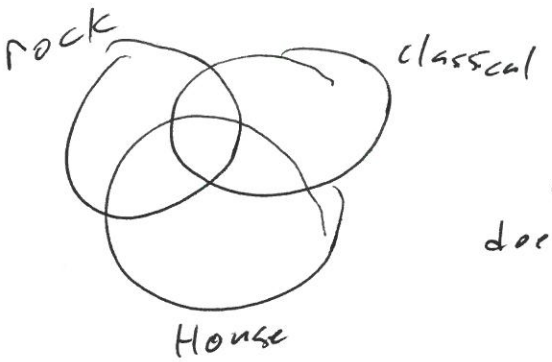
11-06 Warm Up:

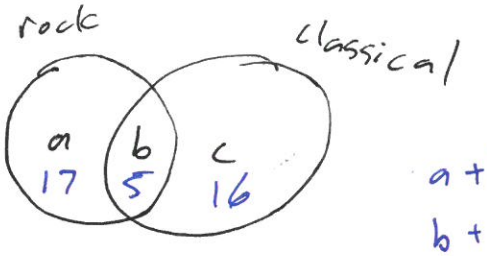
100 students are surveyed on music tastes. 21 like classical music, 22 like rock music, 27 like house music. 5 students like both classical and rock music. How many of those that like rock do not like classical music?

Answer: Those that like rock music minus those who like rock music and classical music

$22 - 5 = 17$



notice that our question does not involve house music



$a + b = 22$

$b + c = 21$

$b = 5$

Decision Algorithms

- ☆ Addition Principle
- ☆ Multiplication Principle.

ex/ Ben & Jerrys 15 ice cream flavors
5 frozen yogurt flavors

Addition

15 + 5 = 20 total options.

ex/ Ben & Jerrys 15 ice cream flavors
3 cone sizes.

Multiplication

15 × 3 = 45 total options

ex/ Put them together:
choosing between 15 ice cream flavors
and 5 frozen yogurt options
and each has 3 options for cone size.
How many total option?

Alternative 1: Ice Cream. } 45 choices
Step 1: flavor 15
Step 2: size 3
Step 3: cup or cone 2

+

Alternative 2: Frozen Yogurt } 15 choices
Step 1: flavor 5
Step 2: size. 3
Step 3: cup or cone 2

=

60 choices. total.

What if we had option
to have our frozen desert
in a cup or a cone!
(cup also has 3 sizes)

$15 \times 3 \times 2 + 5 \times 3 \times 2 = 120$ choices

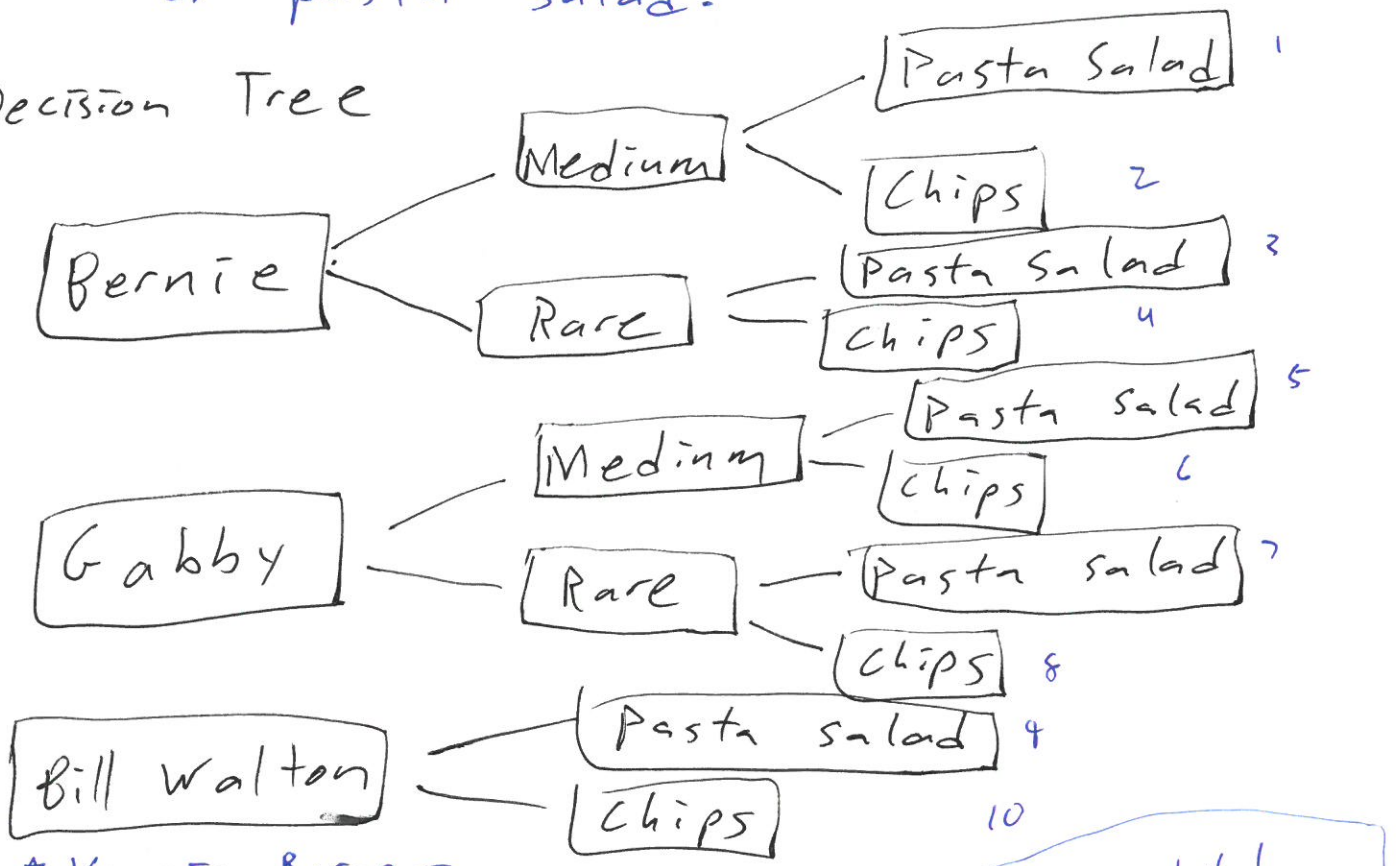
ex/ Getting a Burger at Players Retreat

considering ~~Decided between~~ the Bernie, Gabby and Bill Watson Burger*

Choice between Medium & Rare for cooking.

For side have the option of chips or pasta salad.

Decision Tree



* Veggie Burger

10 total choices.

2 meat options
1 veggie option

all meat options have 2 levels of cooked and 2 sides
Veggie option has 2 sides

$2 \times 2 \times 2 + 1 \times 2 = 8 + 2 = 10$

ex/ You forgot your 4-digit bike lock combination

How many possibilities are there?

- Each didgit can be 0-9

4 places and 10 options for each place

$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$ possibilities.

- We remember that there were no digits used consecutively in our code.

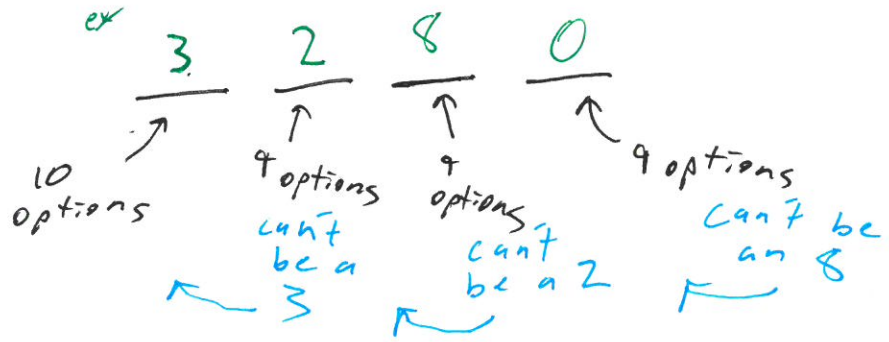
ex/ 1324 is ok

1322 is not ok.

How many options do we have now?

4 digits

ex



$10 \cdot 9 \cdot 9 \cdot 9 = 7{,}290$ possibilities

6.4 Permutations & Combinations

ex You are putting together a playlist for a quick jog and have chosen 5 songs: "Eye of the Tiger"
"Lose Yourself"
"Dog Days are Over"
"Born to Run"
"Edge of Glory"

How many different ways are there to make this playlist?

We have a set of songs, how many ways are there to order them?

ex
1. "Eye of Tiger..."
2. "Lose ..."
3. "Born ..."
4. "Edge ..."
5. "Dog Days ..."

an example of one ordering

We call an ordered list like the one above a <u>permutation</u>

How many permutations of these 5 songs are there?

Lets count them using a decision alg like before:

Step 1:	Choose the first song:	5 options
Step 2:	" = second song:	4 options.
Step 3:	" = third " :	3 options
Step 4:	" = fourth " :	2 options
Step 5:	" = fifth " :	1 option

Total # of permutations:

$$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$$

If we had n songs we wanted to order

$$n \times (n-1) \times (n-2) \times \cdots \times 4 \times 3 \times 2 \times 1 = n!$$

. If we have n things we want to order there are $n!$ ways to order them.

Sometimes we will not want to order all of the items.

ex/ Say we were picking 5 songs from A 100 song library.

How many different playlists can we make? (no repeating)

Step 1:	100 options
Step 2:	99 options
Step 3:	98 options
Step 4:	97 options
Step 5:	96 options

# of options $= 100 \cdot 99 \cdot 98 \cdot 97 \cdot 96$

ex/ You are casting roles for a play of The Crucible.
You have 10 auditions (total) for the characters, Abigail, Elizabeth, Mary, and Rebecca.

How many ways are there to assign roles?

10 people and 4 roles.

Step 1: Cast Abigail — 10 choices
Step 2: Cast Elizabeth — 9 choices
⋮

$10 \cdot 9 \cdot 8 \cdot 7 = 5,040$ ways to cast these roles

How do we write this using factorials?

$$10 \cdot 9 \cdot 8 \cdot 7 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10!}{6!}$$

$$= \frac{10!}{(10-4)!}$$

~~A Permutation~~ The # of permutations of n items taken r at a time is

$$P(n,r) = \frac{n!}{(n-r)!}$$