11-1

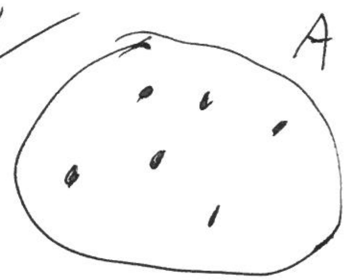
6.2 Cardinality

The cardinality of a set is the size of the set. (i.e. number of elements in the set)

ex/ $A = \{a, b, c\}$ Cardinality of A is 3

$n(A) :=$ Cardinality of A $\quad n(A) = 3$

ex/



$n(A) = 6$

ex/ $n(\emptyset) = 0$

- Small simple sets finding the Cardinality is easy.
- More complex for more complex sets.

★ Cardinality of a Union

$$n(A \cup B) = n(A) + n(B) \; ?$$

ex $A = \{a, b, c\}$ $\quad n(A) + n(D) = 6$

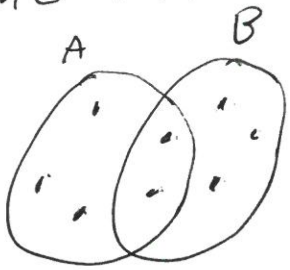
$D = \{b, c, d\}$

$A \cup D = \{a, b, c, d\}$ $\quad n(A \cup D) = 4$

$n(A \cup B) = n(A) + n(B)$ is <u>NOT</u> the right formula!

Problem: we double count the duplicates.

the fix:



$n(A \cup B) = 8$

$n(A) = \cancel{6}\ 5$

$n(B) = 5$

need to subtract the intersection

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

in above $n(A \cap B) = 2$

$5 + 5 - 2 = 8$ ✓

• ~~Cardinality of~~

ex $S = \{\text{set of students in this class}\}$ # 200

$A = \{\text{students with dogs}\}$ # 70

$B = \{\text{students with cats}\}$ # 40

$n(A \cap B) = 6$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 70 + 40 - 6$$
$$= 104$$ students have a cat or dog.

Q: How many have neither cat or dog?

$$n((A \cup B)^c) = ?$$

☆ Cardinality of a complement

- S := universal set

in our example above its the students in class.

$$n(S) - n(A) = n(A^c)$$

note: $n(S) - n(A^c) = n(A)$

set of students without either a cat or dog

$$(A \cup B)^c = A^c \cap B^c$$

$$n((A \cup B)^c) = n(S) - n(A \cup B)$$
$$= 200 - 104 = 96$$

ex In 2011 we have sales data from Apple of their iPods, iPhones, iPads from Q2, Q3, Q4 (in millions)

U', $A \cap U'$, $(A \cap U)'$, $C \cup U$

U' : ~~wh~~ Sales in Q3 & Q4

note: $U' = V \cup W$

$n(U') = 37.1 + 34.8$

$A \cap U'$: Sales of iPods in Q3 & Q4

$n(A \cap U') = 7.5 + 6.6$

$(A \cap U)'$: All sales except iPods in Q2

$$n((A \cap U)') = n(S) - n(A \cap U)$$
$$= 104.3 - 9.0$$

$C \cup U$: Sales in Q2 and ~~iPad~~ total iPad sales

$$n(C \cup U) = n(C) + n(U) - n(C \cap U)$$
$$= 25.1 + 32.4 - 4.7$$

	iPod (A)	iPhone (B)	iPad (C)	Total
(U) 2011 Q2	9.0	18.7	4.7	32.4
(V) 2011 Q3	7.5	20.3	9.3	37.1
(W) 2011 Q4	6.6	17.1	11.1	34.8
Total	23.1	56.1	25.1	104.3

$n(A) = 23.1$ $\quad$ $n(V) = 37.1$

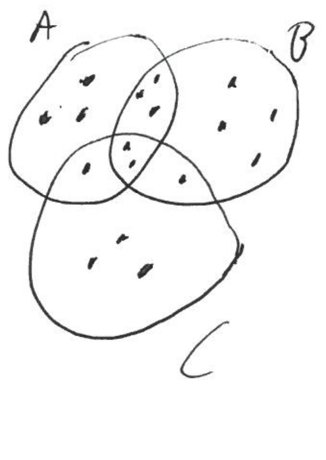
what does $B \cap W$ represent?

- the iPhone sales in Q4 of 2011

$n(B \cap W) = 17.1$

what do these sets describe and what is their cardinality?

Formula for Triple intersections:



$n(A \cup B \cup C) =$ count the dots

$= 17$

what are we counting once, when we $n(A) + n(B) + n(C)$

what are we counting twice?

what are we counting three times?

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

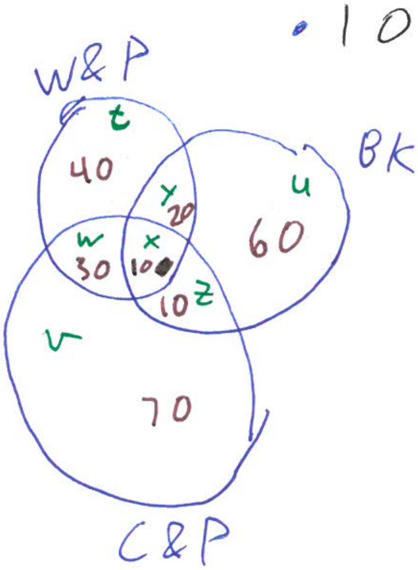
long story short: using venn diagrams is helpful.

ex/ 300 students surveyed about three Russian novels.

- 100 have read War & Peace
- 120 have read Crime & Punishment
- 100 have read The Brothers Karamazov

Also:

- 40 had only read War & Peace
- 70 had read W&P but not BK
- 80 had read BK but not C&P
- 10 have read all 3.



$x = 10$

$t = 40$

$t + w = 70 \Rightarrow w = 30$

$t + w + x + y = 100 \Rightarrow y = 20$

$y + u = 80 \Rightarrow u = 60$

$y + x + u + z = 100 \Rightarrow z = 10$

$v + w + x + z = 120 \Rightarrow v = 70$

who hasn't read any of these books?

$n((W\&P \cup Bk \cup C\&P)^{c}) = ? \quad - \quad = 60$

$n(S) = 300$

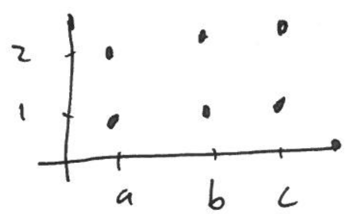
$n(W\&P \cup Bk \cup C\&P) = 240$

Cardinality of Cartesian Product

$A = \{a, b, c\}$

$B = \{1, 2\}$

$A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$



$n(A \times B) = n(A)n(B)$

$n(A \times B \times C) = n(A)n(B)n(C)$

ex

If we toss a coin 2 times in sequence how many outcomes are there?

$S = \{H, T\}$ $\quad n(S \times S) = ?$

$= n(S)n(S) = 2 \cdot 2 = 4$

If we toss a coin 10 times in sequence how many outcomes are there?

$n(S \times S \times S \times S \times S \times S \times S \times S \times S \times S) = ? \quad 2^{10} = 1024$

6.3 Decision Algorithms:
Addition & Multiplication Principles

In a given scenario how many choices do we have?

ex/ You go to the Ben & Jerry's across from Meredith and they have 15 ice cream flavors and 5 frozen yogurt flavors.

How many options?

$$15 + 5 = 20$$

☆ Addition Principle: when choosing from <u>disjoint</u> alternatives <u>add</u> the options together

ex/ 15 ice cream flavors
3 sizes of cones.

How many options do we have?

$3 \cdot 15 = 45$ our set of choices is the cartesian product of A - set of flavors, B - set of sizes

☆ Multiplication Principle: When making a <u>sequence</u> of choices <u>multiply</u> our options