10-30 Chapter 6

6.1 Sets - ~~something that contains~~ is a collection of elements.

$A = \{a, b, c\}$ $\quad a \in A$

$B = \{a, b, c, d\}$ $\quad A \subseteq B$ $\quad A \subset B$

~~$B \subseteq A$~~, ~~$B \subset A$~~

$C = \{A, B\}$ $\quad C \notin A$ $\quad A \in C$

↳ $C = \{\{a, b, c\}, \{a, b, c, d\}\}$ $\quad b \notin C$

ex/ D is the set of sets of 3 ~~elements~~ integers between 0 and 5

- 1, 2, 3, 4 are inbetween 0 & 5
- 3 ~~elements~~ integer sets btwn 0 & 5

$D = \{\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$

Weird things about sets:

- Set that contains everything?
- What about the set that contains all sets?
  → would contain itself.
- The set that contains all sets that contain themselves?

distinguishable dice

we can tell the dice apart. Maybe order matters.

$$T = \left\{ \begin{array}{ccc} (1,1), (1,2), \ldots, (1,6) \\ (2,1), \ldots, (2,6) \\ \vdots \qquad\qquad \vdots \\ (6,1) \ldots \ldots (6,6) \end{array} \right\}$$

set of outcomes has 36 elements in it.

ex rolling two dice and we are interested in the set of outcomes of their sum.

$$T = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

11 different possibilities.

How to write sets via set notation:

$$B = \{0, 2, 4, 6, 8\}$$

$$B = \{n \mid n \text{ is a nonnegative even \# less than } 10\}$$

$$C = \{n \mid n \text{ is a positive odd \# less than one million}\}$$

$$B = \{x \mid \text{"} \qquad \text{"} \}$$

B [is] [the set of] [all x] [such that] [" "]

- The set of all ~~sets~~ sets that do not contain themselves?
  - Does it contain itself. Russels Paradox.

In probability we often want to describe all the possible outcomes from an event

ex/ Set of outcomes from flipping a coin.

$$S = \{H, T\}$$

what about rolling a die? (six-sided die)

$$S = \{1, 2, 3, 4, 5, 6\}$$

rolling two die? distinguishable die or indistinguishable die?

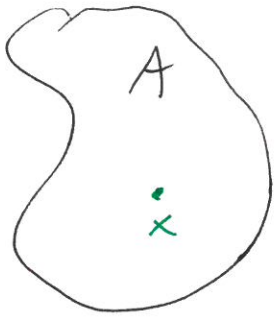
indistinguishable die - order doesn't matter

rolling a (2, 5) is the same as rolling a (5, 2).

$$T = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)\}$$

21 elements in this set of outcomes for two indistinguishable die.

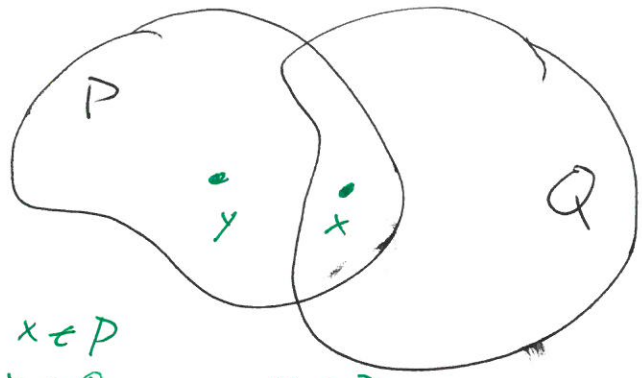
To visualize sets use Blobs/ Venn Diagrams.



$x \in A$

$y \notin A$

$C \subset B$



$x \notin P$

$x \notin Q$

$y \notin P$

$y \notin Q$

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ex/ A is the set of people with a facebook account

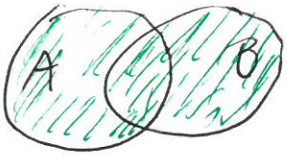
B is the set of people with an instagram account.

How do we describe the set of people with either a facebook & instagram, account or both.

Union of sets: $A \cup B$ is the set of all elements in A or B

set notation: $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

$A \cup B$ ☆ inclusive or - allowed to be in both sets.

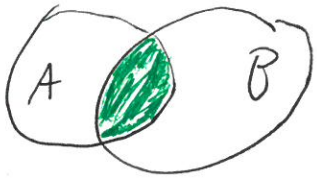


Intersection of sets

People with a facebook account <u>and</u> an instagram account.

$A \cap B$ is the set of elements in A <u>and</u> B

set notation: $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$



$A \cap B$

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ex $A = \{ a, b, c \}$

$B = \{ c, d, f \}$

$A \cup B = \{ a, b, c, d, f \}$

$A \cap B = \{ c \}$

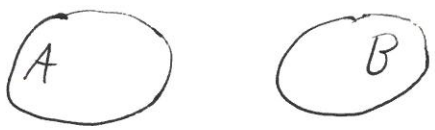
ex $A = \{ 1, 3, 5 \}$

$B = \{ 2, 4, 6 \}$

$A \cup B = \{ 1, 2, 3, 4, 5, 6 \}$

$A \cap B = \{ \} = \emptyset$

If the intersection of two sets is empty then the sets are called <u>disjoint</u>



ex $A \cup \emptyset = \{1, 3, 5\} = A$

$B \cap \emptyset = \{\} = \emptyset$

- What if we want the set of all people without a face book account.

$A = \{x \mid x \text{ is a person with a facebook account}\}$

The Set Complement A'

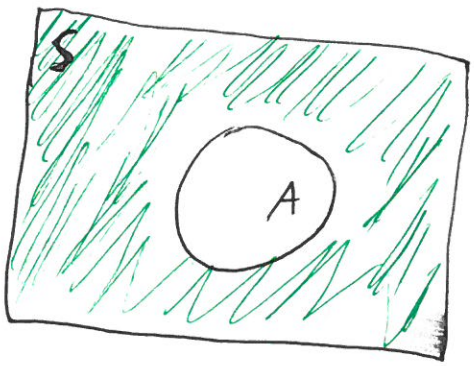
Idea is A' is the set of all elements not in A.

well kinda...

Need a Universal set S

This universal set is the set of everything that ~~[illegible]~~ contains the things we are interested in this problem

In this case S is the set of all people.



A'

$A' = \{x \in S \mid x \notin A\}$

The Cartesian Product

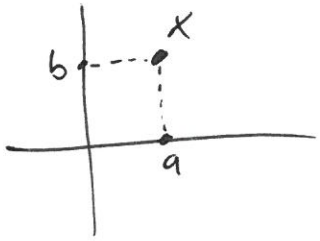
The set of all ordered pairs of two sets.

$$A \times B = \{(a, b) \mid a \in A,\ b \in B\}$$

↑ cartesian product

example $\mathbb{R} \times \mathbb{R}$ is what we graph functions on.

↑ real number



$x \in \mathbb{R} \times \mathbb{R}$

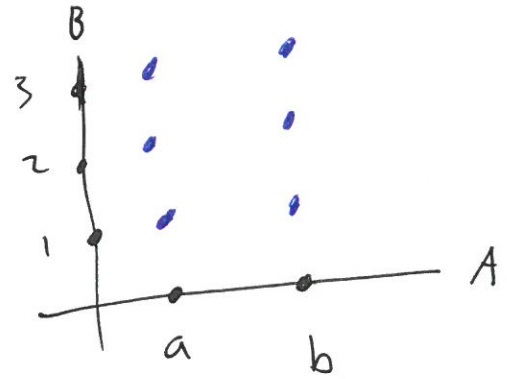
$x = (a, b)$

ex $A = \{a, b\}$

$B = \{1, 2, 3\}$

$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

visualize this



ex $S = \{H, T\}$ what is $S \times S$?

$$S \times S = \{(H,H), (H,T), (T,H), (T,T)\}$$

$S \times S$ is the set of outcomes from flipping two coins.

ex/ $T = \{1, 2, 3, 4, 5, 6\}$ rolling a die

$T \times T$ = set of out comes from rolling two dice.

Q: distinguishable or indistinguishable?

why? - because order matters for Cartesian Product.

De Morgan's Laws:

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$