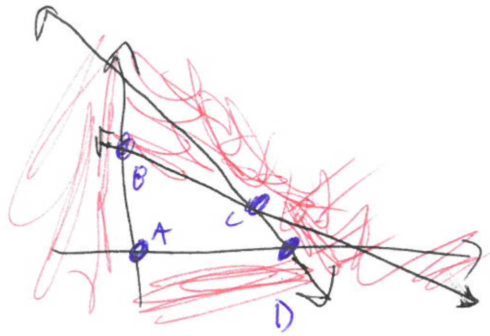
Day 14 10-16

Last time: we had a problem about
Howling Cow Egg Nog
we ended up with a region:



under our original cost function

$$p = .2x + .3y$$

reduced fat

full fat Egg Nog

point C gave the optimal solution.

- This corresponded with using up all of the milk & all of the cream

But! under the cost function: $p = .4x + .2y$

point B gave the optimal solution.

Notice point B does not correspond with using all our resources.
It uses all the milk but not all the cream.

Could be beneficial or reasonable to ask
→ under the optimal solution how much of a resource is left over?

ex/ A bank has \$25 million allocated for home loans.

- Every year they allocate at least \$10 million for luxury condos.
- A government grant requires that they allocate at least a third of their total loan towards low-income affordable housing.

a.) If their return on luxury condos is 12% and the return on affordable housing is 10% how much should they allocate for each type of housing in order to maximize their return?

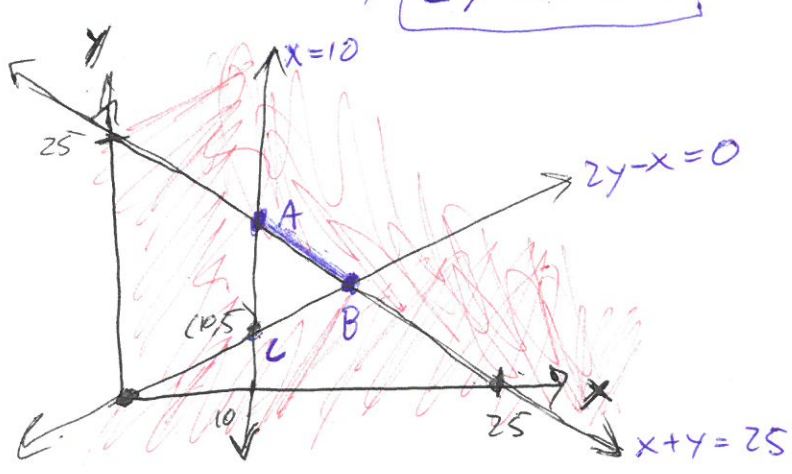
b.) Same as a.) but what if the return on both was 12%?

x = luxury condo allocations
y = affordable housing allocations

- $\boxed{x \geq 10}$
- $\boxed{x + y \leq 25}$
- $y \geq \frac{x+y}{3} \Rightarrow 3y \geq x + y$
  $\Rightarrow \boxed{2y - x \geq 0}$

function to optimize:

$\boxed{.12x + .10y = P}$



$x \geq 10$

$x = 10$

$x + y = 25$

$y = 0 \Rightarrow x = 25 \qquad (25, 0)$

$x = 0 \Rightarrow y = 25 \qquad (0, 25)$

$2y - x = 0$

$y = 0 \Rightarrow -x = 0 \qquad (0, 0)$

$x = 0 \Rightarrow 2y = 0 \qquad (0, 0)$

$x = 10 \Rightarrow 2y - 10 = 0 \qquad (10, 5)$

		$p = .12x + .10y$
A	$(10, 15)$	2.7
B	$\left(\frac{50}{3}, \frac{25}{3}\right)$	$2.8\overline{3}$ ☆
C	$(10, 5)$	1.7

A: intersection of
$x = 10$
$x + y = 25$

B: intersection of
$x + y = 25$
$2y - x = 0$

C: intersection of
$x = 10$
$2y - x = 0$

b.) What if $p = .12x + .12y$

		$p = .12x + .12y$
A	$(10, 15)$	3 ☆
B	$\left(\frac{50}{3}, \frac{25}{3}\right)$	3 ☆
C	$(10, 5)$	1.8

both maximize our p.

What if our feasible region is unbounded?

- We put a box that contains all the corners of our feasible region.
- Evaluate at all corners → including those made by the box.
- If our optimal solution lies on a corner that includes a side of our box
  → No optimal solution

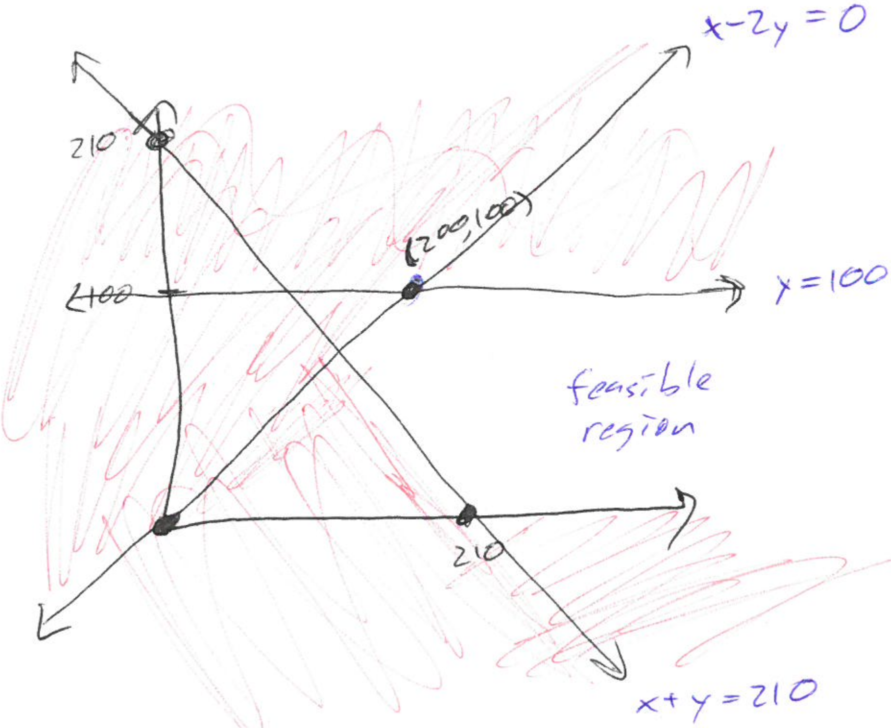
other corners will give optimal solutions if not on the box.

ex/ we want to ~~opt~~ minimize

$$C = 3x + 2y$$

subject to constraints

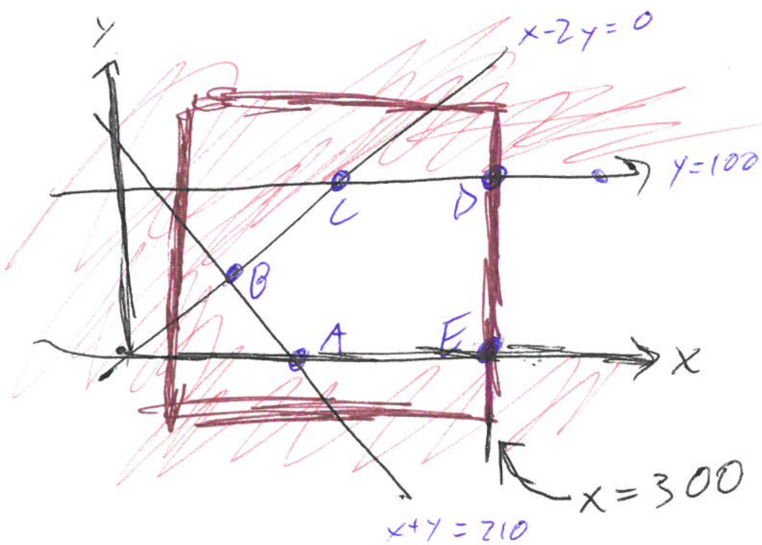
$$x - 2y \geq 0$$
$$y \leq 100$$
$$x + y \geq 210$$
$$x \geq 0, \quad y \geq 0$$



$x - 2y = 0$
(0,0)
(200,100)

$y = 100$

$x + y = 210$
(0,210)
(210,0)



If D or E give optimal solutions
⇒ on the box
⇒ no optimal solution

If A, B, C give optimal solution
⇒ Exactly the optimal solutions

	coordinates	$p = 3x + 2y$
A	(210, 0)	630
B	(140, 70)	560 ☆
C	(200, 100)	800
D	(300, 100)	1,100 ☆
E	(300, 0)	900

minimize → and we get B as optimal solution

maximize → D gives maximal value

but since D touches the box

→ No Optimal solution!

So far we've only looked at 2 varible problems

what happens if we have more than 2 variables?

↑ our dimension.

hard to visualize ≥ 3 dimensions

☆ Use the simplex method.

→ the rest of chapter 5