10-9 Day 12

Warm up: Graph the inequality

$3x + 2y \leq 12$

Solution: 1st step will be to graph the line

$3x + 2y = 12$

There are many ways to do this

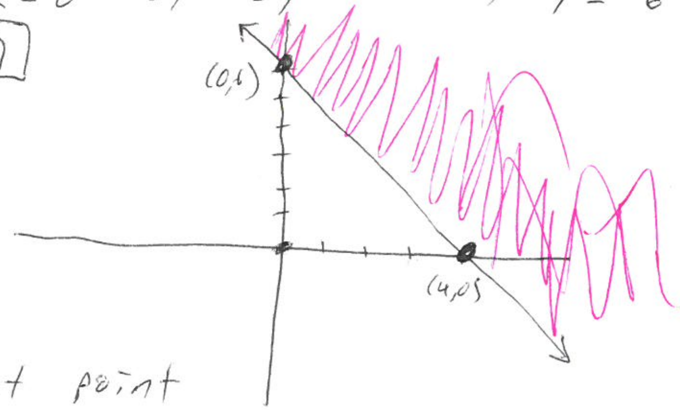
→ but here we will find x & y intercepts.

- set $y = 0 \Rightarrow 3x = 12 \Rightarrow x = 4$

  $\boxed{(4,0)}$

- Set $x = 0 \Rightarrow 2y = 12 \Rightarrow y = 6$

  $\boxed{(0,6)}$

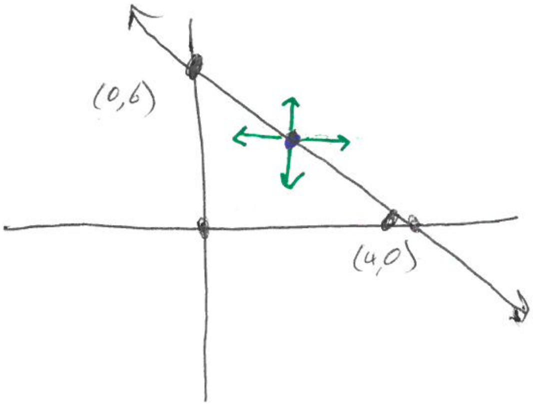


- use test point $(0,0)$ into $3x + 2y \leq 12$

  $0 \leq 12$ ✓

  so $(0,0)$ lies on the solution side

How do we know we can use a test point to determine which side the solutions are on?



what happens when we move a point on the line slightly up & down or left & right.

Remember our inequality $3x + 2y \leq 12$

we have a point where $3x + 2y = 12$

- what happens if we move that point to the ~~right?~~ left?
  - we decrease the value of x.

    So if we had $3x + 2y = 12$

    now moving x ← will give $3x + 2y \leq 12$
- what about moving x to the right?
  - we increase the value of x

    so now $3x + 2y \geq 12$
- what about y up & down
  - $y \uparrow \Rightarrow 3x + 2y \geq 12$
  - $y \downarrow \Rightarrow 3x + 2y \leq 12$

ex/ Graph $3x - 2y \le 6$

find our x & y intercept

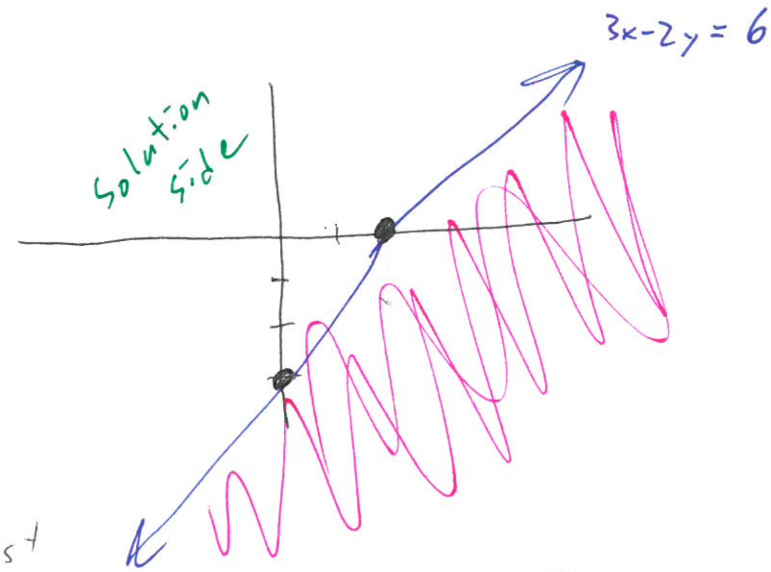
- $x = 0 \Rightarrow$ ~~$-2y \le 6 \Rightarrow$~~

$-2y = 6 \Rightarrow y = -3$

$(0, -3)$

- $y = 0 \Rightarrow 3x = 6 \Rightarrow x = 2$

$(2, 0)$



plug in test point. $(0,0)$

$3(0) - 2(0) \le 6$?

$0 \le 6$ ✓

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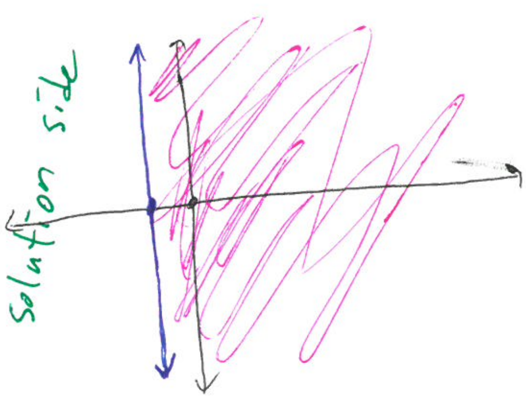
ex/ $x \le -1$

Question
how do we graph $x = -1$



plug in test point $(0,0)$

$0 \le -1$?

No ✗

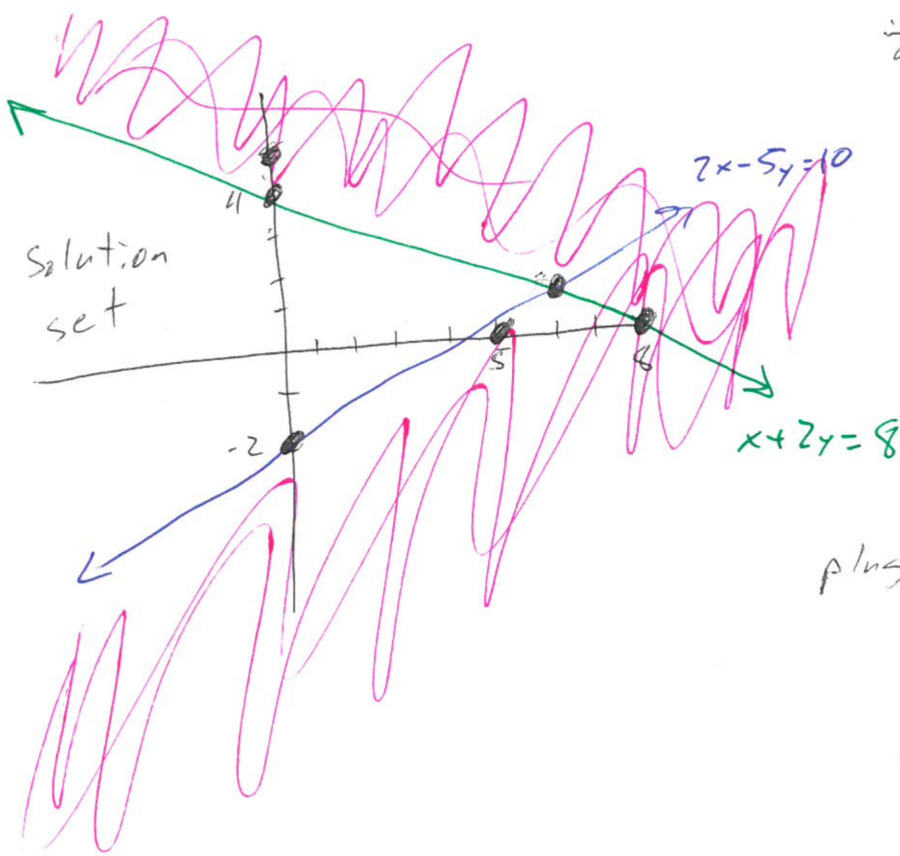
Graph Simultaneous Inequalities:

think systems of inequalities

ex/

$$2x - 5y \le 10$$
$$x + 2y \le 8$$

where are both of these inequalities satisfied?



$2x - 5y = 10$

$\boxed{\begin{matrix}(5,0)\\(0,-2)\end{matrix}}$

$x + 2y = 8$

$\boxed{\begin{matrix}(0,4)\\(8,0)\end{matrix}}$

plug in $(0,0)$

$0 \le 10$ ✓

$0 \le 8$ ✓

In the following section finding corners of our solution set is going to be very important.

So where is the corner in the above example?

Need to find intersection of

$$\begin{cases} 2x - 5y = 10 \\ x + 2y = 8 \end{cases} \qquad \begin{matrix} 2x - 5y = 10 \\ 2x + 4y = 16 \end{matrix} \Rightarrow \quad 9y = 6 \quad \boxed{y = 2/3}$$

$$x + 2\left(\tfrac{2}{3}\right) = 8 \Rightarrow x + \tfrac{4}{3} = 8 \qquad \boxed{x = 8 - \tfrac{4}{3}}$$

ex/

$$3x - 2y \le 6$$
$$x + y \ge -5$$
$$y \le 4$$

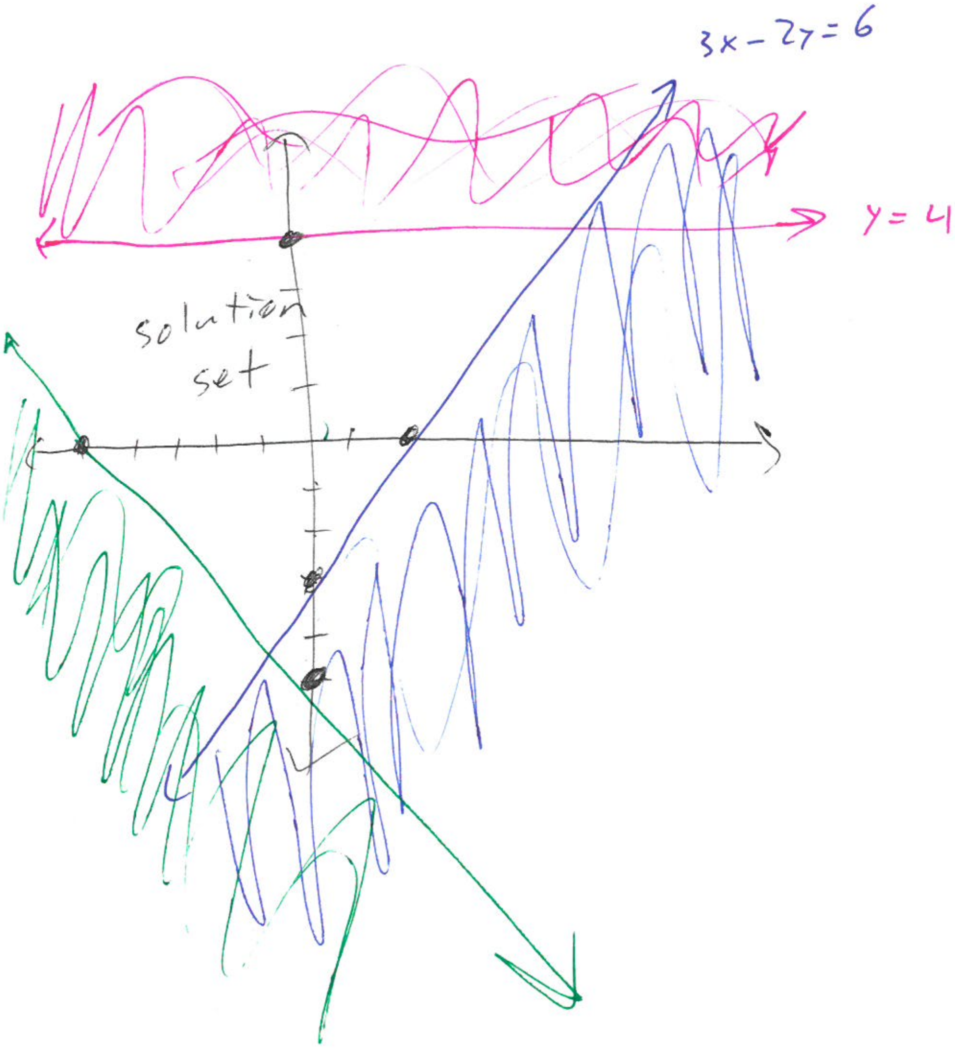
$3x - 2y = 6$

$\boxed{\begin{matrix}(0,-3)\\(2,0)\end{matrix}}$

$x + y = -5$

$(0,-5)$

$(-5,0)$



Recognizing inequalities in word problems:

"at most" $\le$

"up to" $\le$

"no more than" $\le$

"at least" $\ge$

"or more" $\ge$

ex/ You are a sports nerd but also a pokemon nerd
baseball card packs are \$2
pokemon card packs are \$4
you have up to \$35 to spend. How many of each can you buy?

x: # of baseball card packs
y: # of pokemon card packs

$$\boxed{2x + 4y \leq 35}$$

ex/

In 2011 the Bank of Hawaii stock at \$45/share and JP Morgan Chase stock cost \$40/share.
BOH yeilded 4% per year in dividends
JPM yeilded 2.5% per year in dividends

- We have \$25,000 ~~and~~ up to and we want to earn at least \$760 in dividends (assuming the stocks perform the same way)

- Write out the 2 inequalities in here and graph the feasible region (the solution set)
- How many shares of each ~~stock~~ can we buy?

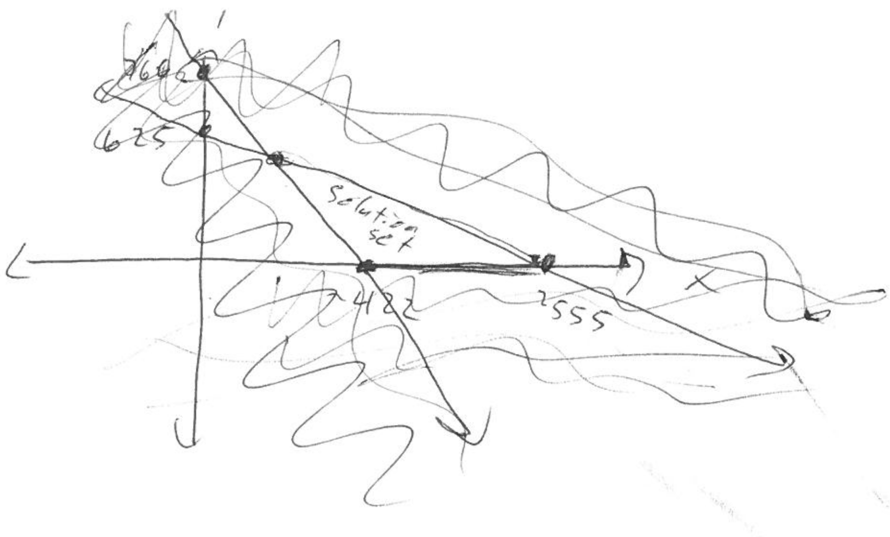
solutions

x: # BOH shares

y: # JPM shares

$$45x + 40y \leq 25{,}000$$

$$(0.04)45x + (0.025)40y \geq 760$$



Section 5.2

Solving Linear Programming Problems Graphically.

→ Given some constraints what is the best option?

If our constraints and objective function are linear this is a linear programming problem.

__Objective function:__ ~~the~~ A function that represents the quantity we are trying to optimize (make as large/small as possible)

· Here we will focus on two unknowns so objective functions will be of the form

$$ax + by$$

under constraints (any # of these)

$$cx + dy \le e$$

$$cx + dy \ge e$$

Fundemental Theorem of linear Programming:

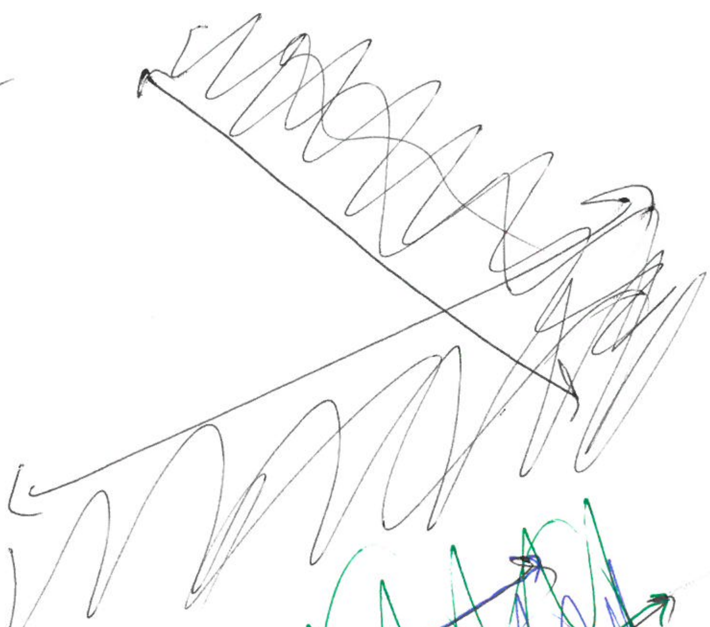
- If a LP problem has an optimal solution - at least one occurs at a corner of the solution set (feasible region)
- Linear Programming Problems with bounded, non-empty feasible regions <u>always</u> have an optimal solution.

Bounded? We can put a box around the solution set

ex

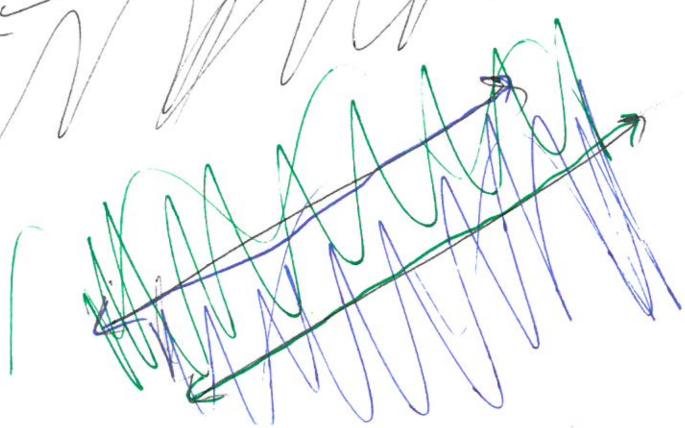
is bounded and non-empty

ex



is not bounded. and non-empty

ex



empty solution set